

# Deformation Data Recovery Based on Compressed Sensing in Bridge Structural Health Monitoring

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## ABSTRACT

Full life-cycle and real-time structural health monitoring (SHM) rely on numerous, heterogeneous sets of data collected from sensors and extracting features that support the estimation of the health condition of a bridge. It is the principal challenges facing the SHM application on transmission and storing such huge amounts of data. Compressive sensing (CS) is a novel data acquisition method whereby the compression is done in a sensor simultaneously with the sampling. In this work, we established the possibility of compressed sensing to address this challenges on bridge SHM.

Based on the sparsity of the deformation data of bridge, we proposed a random sampling scheme based on CS to minimize the number of field data. The CS recovery performance is mostly determined by the decomposition basis which is associated with the sparsity of the sampling signal. Different bases have been tested to recover the deformation data. Experimental results demonstrate the proposed method allows a reduction of the measurement data with an acceptable recovery accuracy. And reconstruction performance based on DWT to sparse transform is better than that based on DCT to sparse transform. When the compression ratio is above 0.6, the reconstruction error grows moderately; whereas the reconstruction error grows rapidly as the compression ratio is below 0.6. With the compression ratio decreased from 0.6 to 0.2, reconstruction error is reduced about 2.5 times by using DWT and reduced about 2 times by using DCT.

## INTRODUCTION

Recently, many bridge collapses occurred. The collapses of the bridges have been a major risk in the world, causing serious physical and psychological harm to the people of disaster. Reducing the bridge collapse accident is the urgent need to solve the problem.

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One of the best ways to reduce the bridge collapse accident is real-time monitoring of bridge health. There are numerous methods can real-time detect the health status of bridge at present. E.g., deformation-based SHM, temperature-based SHM, vibrated-based SHM [1-2].Continued collection of corresponding parameters is a major challenge for real-time health monitoring. With the development of wireless communication, wireless sensor network (WSN) can collect data for 24 hours without interruption. However, such a large data capacity will certainly bring great pressure on the WSN. In view of this shortcoming, people have been exploring new methods to reduce the capacity of data collection and transmission, thereby reducing the damage to the WSN and improving the efficiency of bridge SHM.

Traditionally, the data compression method is first to collect the data and then compress it. Although the method can reduce the capacity of the transmission data, it does not decrease the capacity of the collected data. To address this drawback, Candès proposed a method of simultaneous sampling and compression, called compressed sensing (CS) [3].Since this technology is first presented, it immediately aroused the interest of the researchers. The issue of CS has been extensively used in various fields. In the smart grid, the energy signal was compressed and reconstructed by using CS [4]. In electronic nose system, Djelouat deals the CS to reduce the storage capacity of the sensor and used the distributed compressed sensing (DCS) to ensure the reconstruction precision [5]. In addition, the CS was applied to the bridge acceleration signal to realize the compression sampling and reconstruction of the signal in [6]. And the data of the WSN is compensated by the CS theory [7]. Nevertheless, CS is seldom in the application of bridge deformation data. In the present paper, we apply this technique to compress sampling and reconstruct the bridge deflection signal that is one of the most important parameters in deformation monitoring. We first use discrete wavelet transform (DWT) and discrete cosine transform (DCT) to verify the sparsity of bridge deflection data. Then, compression sampling of bridge deflection signals by using random Gaussian matrix. Finally, the received data are reconstructed by orthogonal matching pursuit (OMP).

The remainder of this paper is as follows. The CS theory is used to compression sampling the bridge deflection signal in section 2. Section 3 presents the reconstruction method to recovery the incomplete bridge deflection signal. In section 4, we show the experimental result, and section 5 concludes the paper.

## COMPRESSION SAMPLING THE BRIDGE DEFLECTION SIGNAL

CS model is utilized to compression sampling the bridge deflection signal in this section. Let  $x$  represents the bridge deflection signal,  $x \in R^{n \times 1}$ ,  $\Phi$  denotes the observation matrix,  $\Phi \in R^{m \times n}$  ( $m < n$ ). Taking into account the characteristics of the bridge deflection signal without prior knowledge, the paper chooses random Gaussian matrix as observation matrix  $\Phi$ . So we can obtain the compressed bridge deflection signal  $y \in R^{m \times 1}$  via  $\Phi$ .As shown in Eq. (1) [1]

$$y = \Phi x \quad (1)$$

One of the key conditions to use CS is the signal satisfy sparse performance [1, 8], hence the first step is to test the sparsity of the bridge deflection signal. In this paper, bridge deflection data of 1024 were chosen for processing, as shown in Figure 1.

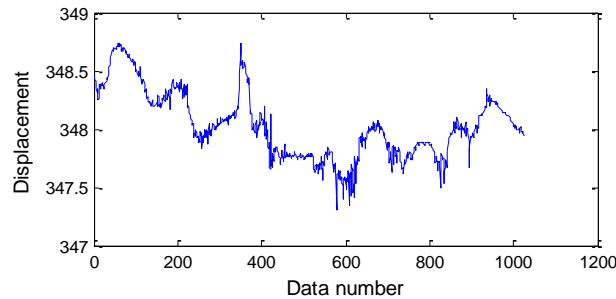


Figure 1. Bridge deflection data

Figure 1 illustrates that the bridge deflection signal is not sparse in the time domain. It is necessary to make it sparse in a specific domain, and it can be converted into a sparse signal by using Eq. (2)

$$\theta = \Psi x \quad (2)$$

where  $\Psi$  is the basis matrix,  $\Psi \in R^{n \times n}$  [1,8]. Fourier transform, discrete cosine transform (DCT) and wavelet transform (WT) are three familiar sparse transformations. WT can be divided into discrete wavelet transform (DWT) and continuous wavelet transform (CWT). Considering the time discretization of the bridge deflection signal, DWT and DCT was used to test the sparsity of the bridge deflection signal.  $\theta$  is the decomposition coefficients vector,  $\theta \in R^{n \times 1}$ . The sparse transformation diagram is shown in Figure 2.

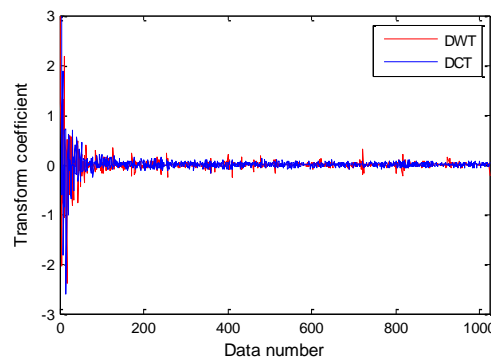


Figure 2. Sparse data in transform domain

Figure 2 shows that the sparsity of the bridge deflection signal in the DWT domain is greater than that in the DCT domain. The different sparsity of the different domain

will lead to the difference of the reconstruction performance, which is shown in the section4.

Substituting Eq. (2) into Eq. (1), another expression of the CS model is shown in Eq. (3)

$$y = \Phi\Psi^{-1}\theta = A\theta \quad (3)$$

where  $A$  represents the sensing matrix,  $A \in R^{m \times n}$  [1,8].

## RECONSTRUCTION THE BRIDGE DEFLECTION SIGNAL

This section will recovery the compression bridge deflection signal. To achieve  $\theta$  from Eq. (3). The optimization problem can be described as [8]

$$\hat{\theta} = \arg \min \|\theta\|_0 \quad \text{s.t.} \quad y = A\theta \quad (4)$$

where  $\hat{\theta}$  denotes the reconstruction of decomposition coefficients vector [1,8]. Because  $A \in R^{m \times n}$ ,  $x \in R^{n \times 1}$ ,  $m < n$ . So there are  $m$  equations and  $n$  unknowns, this is an underdetermined equation. Tao and Candès verified the sparse signal can be recovered when  $A$  satisfies the RIP, as shown in Eq. (4) [8]

$$(1 - \sigma_r) \|\theta\|_2^2 \leq \|A\theta\|_2^2 \leq (1 + \sigma_r) \|\theta\|_2^2 \quad (5)$$

where  $\sigma_r$  is the isometry constants. Baraniuk proposed the equivalent condition of the RIP is the irrelevance between  $\Phi$  and  $\Psi$ , and proved that the Gaussian stochastic measurement matrix satisfies the RIP properties [9]. So  $\theta$  can be restored by solving the Eq. (4). Reconstruction of CS has always been a hot topic of research. Various researchers have been committed to the reconstruction of algorithms. The paper chooses the OMP that performance is more stable to reconstruct the bridge deflection signal.

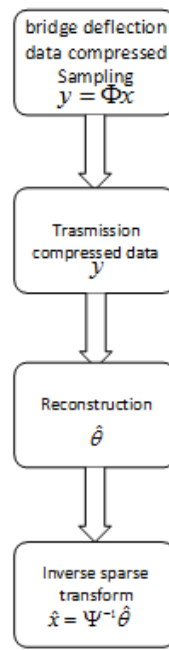


Figure 3. Flow diagram

Thus, the bridge deflection signal compression sampling and reconstruction by the CS model can be described as Figure 3. At the signal acquisition side, the random Gaussian matrix  $\Phi_{m \times n}$  is implanted into the sensor. And the bridge deflection signal is compressed sampling by  $\Phi_{m \times n}$ . Then the compression bridge deflection data streams  $y_{m \times 1}$  is transmitted. Next, at the receiving end, we use the received data  $y_{m \times 1}$  to reconstruct  $\hat{\theta}$  by the algorithm of OMP. Finally, the bridge deflection signal  $\hat{x}$  is restored by an inverse transform sparse data  $\hat{\theta}$ .

## EXPERIMENT RESULTS

In order to verify the feasibility of the algorithm in bridge deformation SHM, the paper uses bridge deflection signals as the research object. It is the vertical displacement of the bridge girder and can reflect the safety status of the whole bridge. Data collected via the connected tube photoelectricity liquid level deflection monitoring system. The system was installed in Chongqing Caiyuanba Yangtze River Bridge, 30 photoelectricity level sensors were arranged in the 15 sections of the main beam. A series of signals processing are run under Matlab 2010(32bit) on a PC with an Intel 2.90 GHZ and 4GB RAM.

Relative error is used as a measure to evaluate the reconstruction performance in this paper, the relative error is written as

$$r = \frac{\|x - \hat{x}\|_2}{\|x\|_2} \quad (6)$$

where  $x$  represents the original signal,  $\hat{x}$  represents the reconstruction error and  $r$  is the relative error. The smaller the reconstruction error, the better the reconstruction.

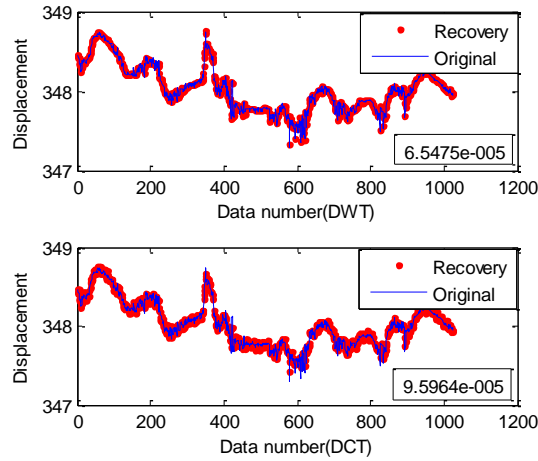


Figure 4. Signal recovery

Figure 4 indicates that the reconstruction effect of bridge deflection signal. As can be seen from the Figure 4, the reconstruction performance that DWT-based transformation is better than DCT-based transformation, the reason is the sparsity of the bridge deflection signal in the DWT domain is greater than that in the DCT domain. The visual representation of more reconstruction performance is shown in Figure 5.

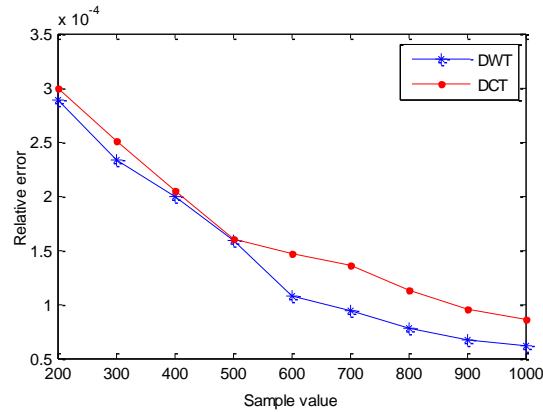


Figure 5. Reconstruction error

Figure 5 reveals that the relationship between the sampling rate and the reconstruction error for the bridge deflection signal. The horizontal axis represents the dimension of the observation matrix and the vertical axis represents the reconstruction error. Obviously, the reconstruction performance based on DWT to sparse transform is preferable to that based on DCT to sparse transform. Our results also illustrate that the reconstruction error increased as the decreased of the sampling rate. When the sample value is above 600(compression ratio about 0.6), the reconstruction error grows slightly; on the contrary, the reconstruction error grows rapidly as the sample value is

below 600. With the compression ratio decreased from 0.6 to 0.2, the error reconstruction is reduced about 2.5 times by using DWT transform and reduced about 2 times by using DCT transform. So reducing the dimension of the observation matrix is actually at the expense of the reconstruction error.

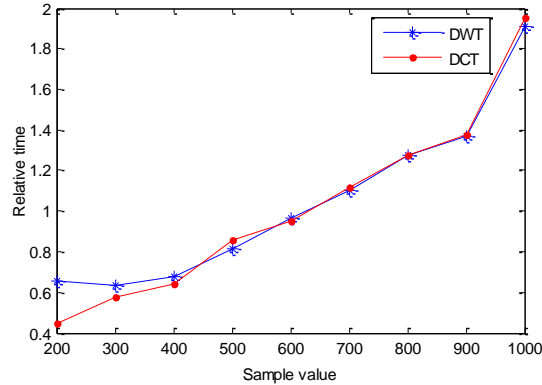


Figure 6. Reconstruction time

Figure 6 illustrates that the relationship between the sampling rate and the reconstruction time for the bridge deflection signal. The horizontal axis represents the dimension of the observation matrix and the vertical axis represents the reconstruction time. It is quite clear that the reconstruction time decreased with the reduced of the compression ratio.

## CONCLUSION

CS model is applied to the compression sampling and reconstruction of the bridge deflection signal in this paper. The experimental results show that the CS scheme can reconstruct the bridge deflection signal with high accuracy when reducing the measurement data; the reconstruction time decreases as the reduction of the sampling rate, while the reconstruction error also increases. In other words, the reduction in storage capacity and the reduction of the reconstruction time are actually at the expense of increasing the reconstruction error. Compression sampling and reconstruction using this method can improve the efficiency of bridge SHM by reducing storage capacity and reconstruction time, but it should be within the range of acceptable reconstruction errors.

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