COVER SHEET

Title: Decision Making for Reference-Free Damage Detection

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ABSTRACT

This paper concerns a reflection on a new decision boundary technique devoted for baseline-free damage detection purpose. Its scope focuses in studying analytically the impact of an unknown disturbance on the behavior of a monitored structure, through linear and nonlinear perturbation models. These perturbation models are introduced to interpret how an unknown disturbance away linearly and nonlinearly a monitored structure from its initial state, which is a healthy one, to its current state, which can be damaged or a healthy one that has undergone environmental/operational variations. To quantify the amount of that unknown disturbance effect, matrix perturbation theory is addressed to define two analytical bounds. The gap between them gives the decision regarding the monitored structure current state. The effectiveness of the proposed approach is demonstrated through an experimental setup of a test coupon aluminum plate, which has undergone varying loads and temperatures conditions, and crack damage cases.

INTRODUCTION

The big concern in reference-free damage detection is to build without any reference database, a decision making (threshold) able to separate between disturbances that come from environmental/operational variations of the healthy state and that come from damaged sate.

The development of this essay was motivated by this challenge, *i.e.* establishing a decision making devoted for reference-free. It focuses on studying the disturbances that a monitored structure undergoes using tools of linear algebra, mainly matrix perturbation theory (MPT).

Pioneering works have addressed the MPT for matrix computation purpose. Based on an *additive perturbation model*, Davis and Kahan [1] have defined an upper bound

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to the rotation of eigenvectors. Based on that model, Wedin [2] has extended Davis and Khan work for the singular subspaces deflection. While Li [3] has conducted studies in MPT through a multiplicative perturbation model.

Regarding SHM, the tool of MPT was introduced by Hajrya and Mechbal [4;5], where the purpose of their works was the development of an analytical threshold based on an additive perturbation model without any assumption of a probabilistic model, but necessitating a reference database.

The reflection that we are seeking to construct in that essay is: how can we guide matrix perturbation theory (MPT) for reference-free decision making?

The word perturbation sounds good in online SHM. It is a disturbance that can comes from two types: (*i*) the environmental/operational variations of the healthy state, such as temperature and loads variations, (*ii*) the presence of damage. Furthermore, as we are in a reference-free context, the aforementioned disturbances are *a priori* unknown. This implies that there is no reason to stipulate that an unknown disturbance has a *linear or a nonlinear impact* on the behavior of a monitored structure. However, there is a reason to assume *blindly* that an unknown disturbance has a linear and a nonlinear impact on the monitored structure. This assumption means that a disturbance causes the behavior of a monitored structure to jump from an initial sate to a new one following a linear and a nonlinear perturbation model. In other words, an unknown disturbance can away linearly (AL) and nonlinearly (ANL) a monitored structure from an initial behavior to a new one.

MPT comes into play to offer us the tools needed to evaluate the AL and ANL concepts. Indeed, MPT allows us to define two analytical bounds that quantify the disturbance effect with well-defined values. These bounds are driven by establishing linear and nonlinear perturbation models of a feature in matrix form. The gap between these bounds gives the decision regarding the current state of the structure, *i.e.* damaged state or a healthy one that has undergone environmental/operational variations.

The layout of this essay is as follows: in section 2, matrix perturbation theory is addressed to drive the SHM decision making in a reference-free context. In section 3, the experimental setup is presented. Section 4 explores the proposed approach on these data. Summary and future research directions are drawn in section 5.

MATRIX PERTURBATION THEORY FOR SHM DECISION MAKING

Problem Statement

Let $\mathbf{A} \in \mathbb{R}^{n_y \times n_y}$ be a feature in matrix form, obtained from n_y data sensors of a monitored structure.

For instance, let us assume that we can obtain a second feature $\widetilde{\mathbf{A}} \in \mathbb{R}^{n_y \times n_y}$ from the same sate. These matrices are extracted from a Blind Separation Source (BSS) technique. For more details about the extraction technique, the reader is invited to read [4].

Based on the same state condition, the presence of a disturbance is seen as a change in the intrepid matrix $\mathbf{A}$ (which reflects the initial behavior of the structure) to matrix $\widetilde{\mathbf{A}}$ (which reflects the new behavior). Accordingly, the singular subspaces of $\widetilde{\mathbf{A}}$ are deflected to those of $\mathbf{A}$. To quantify this deflection, an index of perturbation is proposed, and it is based from the so-called $\sin\theta$ theorem, attributed to Davis and Kahan [1] in its classical form, and extended by Wedin [2].

To drive the proposed index of perturbation (noted IP), we follow the work of [2], where a singular value decomposition (SVD) is used:

$$\mathbf{A} = \mathbf{U\Gamma V} = [\mathbf{U}_1 \quad \mathbf{U}_2]\begin{bmatrix}\mathbf{\Gamma}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{\Gamma}_2\end{bmatrix}[\mathbf{V}_1 \quad \mathbf{V}_2]^T = \mathbf{A}_1 + \mathbf{A}_2 \tag{1}$$

$$\widetilde{\mathbf{A}} = \widetilde{\mathbf{U}}\widetilde{\mathbf{\Gamma}}\widetilde{\mathbf{V}} = [\widetilde{\mathbf{U}}_1 \quad \widetilde{\mathbf{U}}_2]\begin{bmatrix}\widetilde{\mathbf{\Gamma}}_1 & \mathbf{0}\\ \mathbf{0} & \widetilde{\mathbf{\Gamma}}_2\end{bmatrix}[\widetilde{\mathbf{V}}_1 \quad \widetilde{\mathbf{V}}_2]^T = \widetilde{\mathbf{A}}_1 + \widetilde{\mathbf{A}}_2 \tag{2}$$

Let:
$R\{\widetilde{\mathbf{A}}_1\}, R\left\{\widetilde{\mathbf{A}}_1{}^T\right\}$ be the ranges of matrix $\widetilde{\mathbf{A}}_1$, and $R\{\mathbf{A}_1\}$ and $R\{\mathbf{A}_1{}^T\}$ be the ranges of matrix $\mathbf{A}_1$.
$\mathbf{P}_{\mathrm{R}\{\widetilde{\mathbf{A}}_1\}}$, $\mathbf{P}_{R\left\{\widetilde{\mathbf{A}}_1{}^T\right\}}$, $\mathbf{P}_{R\{\mathbf{A}_1\}}$ and $\mathbf{P}_{R\{\mathbf{A}_1{}^T\}}$ be the orthogonal projection on theses ranges, defined as:

$$\mathbf{P}_{\mathrm{R}\{\widetilde{\mathbf{A}}_1\}} = \widetilde{\mathbf{U}}_1\widetilde{\mathbf{U}}_1{}^T, \mathbf{P}_{R\left\{\widetilde{\mathbf{A}}_1{}^T\right\}} = \widetilde{\mathbf{V}}_1\widetilde{\mathbf{V}}_1{}^T \tag{3}$$

$$\mathbf{P}_{R\{\mathbf{A}_1\}} = \mathbf{U}_1\mathbf{U}_1{}^T, \mathbf{P}_{R\{\mathbf{A}_1{}^T\}} = \mathbf{V}_1\mathbf{V}_1{}^T \tag{4}$$

$\|\sin\underline{\boldsymbol{\theta}}[R\{\widetilde{\mathbf{A}}_1\}, R\{\mathbf{A}_1\}]\|_{\mathrm{UI}}$ be the sinus angle between the range $R\{\widetilde{\mathbf{A}}_1\}$ and $R\{\mathbf{A}_1\}$,
$\|\sin\underline{\boldsymbol{\varphi}}[R\left\{\widetilde{\mathbf{A}}_1{}^T\right\}, R\{\mathbf{A}_1^T\}]\|_{\mathrm{UI}}$ be the sinus angle between the range $R\left\{\widetilde{\mathbf{A}}_1{}^T\right\}$ and $R\{\mathbf{A}_1^T\}$.
In this paper, $\|.\|_{\mathrm{UI}}$ denotes a general unitarily invariant norm, and the Frobenius norm $\|.\|_F$ is used for the calculus.
The sinus angle norm of the aforementioned ranges is defined as [4]:

$$\mathrm{IP}_1 = \|\sin\underline{\boldsymbol{\theta}}[R\{\widetilde{\mathbf{A}}_1\}, R\{\mathbf{A}_1\}]\|_F = \|(\mathbf{I}_{n_y} - \mathbf{P}_{R\{\mathbf{A}_1\}})\mathbf{P}_{R\{\widetilde{\mathbf{A}}_1\}}\|_F \tag{5}$$

$$\mathrm{IP}_2 = \|\sin\underline{\boldsymbol{\varphi}}[R\left\{\widetilde{\mathbf{A}}_1{}^T\right\}, R\{\mathbf{A}_1^T\}]\|_F = \|(\mathbf{I}_{n_y} - \mathbf{P}_{R\{\mathbf{A}_1^T\}})\mathbf{P}_{R\left\{\widetilde{\mathbf{A}}_1{}^T\right\}}\|_F \tag{6}$$

Proposal: Index of perturbation
Consider an unknown disturbance that drives the behavior of a monitored structure from an initial behavior to a new one. The change occurred is quantified by the following index of perturbation:

$$\mathrm{IP} = \frac{\sqrt{\mathrm{IP}_1 + \mathrm{IP}_2}}{n_r} \tag{7}$$

where n_r is the number of principal components retained in the studied matrices.

The villain disturbance has let our intrepid matrix $\mathbf{A}$ to $\widetilde{\mathbf{A}}$, and the occurred change has been quantified by the index of perturbation IP. As we are in a reference-free context, the IP is not sufficient to assert the perturbation source, *i.e.* presence of damage or just environmental/operational variations of the healthy state. However, the context of reference-free offers us the possibility to assume blindly that an unknown disturbance causes the behavior of the monitored structure to jump from an initial sate (reflected by matrix $\mathbf{A}$) to a new one (reflected by matrix $\widetilde{\mathbf{A}}$), following *a linear and a nonlinear perturbation models*. In other words, an unknown disturbance can away linearly (AL) and nonlinearly (ANL) a monitored structure from an initial behavior to a new one.

The following subsections focuse on quantifying the concepts AL and ANL, by introducing perturbation models and analytical bounds to IP.

Analytical Bound Models

Defining a bound begins by establishing a model that relates matrix $\widetilde{\mathbf{A}}$ to $\mathbf{A}$. The first way to address that model is to see the relation between $\mathbf{A}$ and $\widetilde{\mathbf{A}}$ as linear:

$$\widetilde{\mathbf{A}} = \mathbf{A} + \delta\mathbf{A} \tag{8}$$

where the matrix $\delta\mathbf{A}$ describes the variation that matrix $\mathbf{A}$ is subjected due to an unknown disturbance, and the relation defined in Eq. (8) is called *the additive perturbation model* [4].

The second way is to relate $\widetilde{\mathbf{A}}$ to $\mathbf{A}$ by a nonlinear model, and one of nonlinear model is the so-called *multiplicative perturbation model* [3]:

$$\widetilde{\mathbf{A}} = \mathbf{D}_1\mathbf{A}\mathbf{D}_2 \tag{9}$$

Analytical Bound using Additive Perturbation Model

The first proposed analytical bound is based on an additive perturbation model, and it is driving from the following:
Recall that the SVD of matrix $\mathbf{A}$ and $\widetilde{\mathbf{A}}$ can be written as follow:

$$\mathbf{A} = \mathbf{U\Gamma V} = [\mathbf{U}_1 \quad \mathbf{U}_2]\begin{bmatrix}\mathbf{\Gamma}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{\Gamma}_2\end{bmatrix}[\mathbf{V}_1 \quad \mathbf{V}_2]^T = \mathbf{A}_1 + \mathbf{A}_2 \tag{10}$$

$$\widetilde{\mathbf{A}} = \widetilde{\mathbf{U}}\widetilde{\mathbf{\Gamma}}\widetilde{\mathbf{V}} = [\widetilde{\mathbf{U}}_1 \quad \widetilde{\mathbf{U}}_2]\begin{bmatrix}\widetilde{\mathbf{\Gamma}}_1 & \mathbf{0}\\ \mathbf{0} & \widetilde{\mathbf{\Gamma}}_2\end{bmatrix}[\widetilde{\mathbf{V}}_1 \quad \widetilde{\mathbf{V}}_2]^T = \widetilde{\mathbf{A}}_1 + \widetilde{\mathbf{A}}_2 \tag{11}$$

Following these decompositions, define the following residual matrices:

$$\mathbf{R}_\mathrm{R} = \mathbf{A}\widetilde{\mathbf{V}}_1 - \widetilde{\mathbf{U}}_1\widetilde{\mathbf{\Gamma}}_1 = -\delta\mathbf{A}\widetilde{\mathbf{V}}_1 \tag{12}$$

$$\mathbf{R}_\mathrm{L} = \mathbf{A}^T\widetilde{\mathbf{U}}_1 - \widetilde{\mathbf{V}}_1\widetilde{\mathbf{\Gamma}}_1^{\,T} = -(\delta\mathbf{A})^T\widetilde{\mathbf{U}}_1 \tag{13}$$

and evaluate their norms, *i.e.*:

$$\|\mathbf{R}_\mathrm{R}\|_F = \left\|\delta\mathbf{A}\widetilde{\mathbf{V}}_1\right\|_F \tag{14}$$

$$\|\mathbf{R}_\mathrm{L}\|_F = \|(\delta\mathbf{A})^T\widetilde{\mathbf{U}}_1\|_F \tag{15}$$

Assume now, that there exist a scalar $\eta > 0$, such that:

$$\eta = \min_{\substack{\tilde{\mu}\in\tilde{\boldsymbol{\Gamma}}_2 \\ \boldsymbol{\lambda}\in\boldsymbol{\Gamma}_1}} |\tilde{\mu} - \lambda| \tag{16}$$

Wedin [2] has proved through a theorem that the index of perturbation defined in Eq.(7) satisfies the following:

$$\text{IP} \leq \frac{\sqrt{\|\mathbf{R}_\text{L}\|_F + \|\mathbf{R}_\text{R}\|_F}}{\eta} \tag{17}$$

where the quantity $\frac{\sqrt{\|\mathbf{R}_\text{L}\|_2 + \|\mathbf{R}_\text{R}\|_2}}{\eta}$ is the upper bound of IP.

For easy reading, that bound is noted as:

$$\beta_\text{A} = \frac{\sqrt{\|\mathbf{R}_\text{L}\|_F + \|\mathbf{R}_\text{R}\|_F}}{\eta} \tag{18}$$

This bound indicates the amount that a disturbance *away linearly* (AL) a monitored structure from an initial behavior to a new one. The second proposed analytical bound is based on nonlinear perturbation model.

Analytical Bound Based on Multiplicative Perturbation Model

Following the nonlinear perturbation model, Li [3] has proved that the index of perturbation satisfies the following:

$$\text{IP} \leq \sqrt{\left\|\left(\mathbf{I}_{n_y} - \mathbf{D}_1{}^T\right)\mathbf{U}_1\right\|_F^2 + \left\|\left(\mathbf{I}_{n_y} - \mathbf{D}_2{}^T\right)\mathbf{V}_1\right\|_F^2} + \frac{\sqrt{\|\{\mathbf{D}_1 - (\mathbf{D}_1^{-})^T\}\mathbf{U}_1\|_F^2 + \left\|\{\mathbf{D_2}^T - (\mathbf{D}_2^{-})^T\}\mathbf{V}_1\right\|_F^2}}{\kappa} \tag{19}$$

where κ is defined as:

$$\kappa = \min_{\substack{\tilde{\mu}\in\tilde{\boldsymbol{\Gamma}}_2 \\ \boldsymbol{\lambda}\in\boldsymbol{\Gamma}_1}} \frac{|\tilde{\mu} - \lambda|}{|\tilde{\mu}|} \tag{20}$$

For easy reading, we note:

$$\beta_\text{M} = \sqrt{\left\|\left(\mathbf{I}_{n_y} - \mathbf{D}_1{}^T\right)\mathbf{U}_1\right\|_F^2 + \left\|\left(\mathbf{I}_{n_y} - \mathbf{D}_2{}^T\right)\mathbf{V}_1\right\|_F^2} + \frac{\sqrt{\|\mathbf{D}_1 - (\mathbf{D}_1^{-})^T\mathbf{U}_1\|_F^2 + \left\|\mathbf{D_2}^T - (\mathbf{D}_2^{-})^T\mathbf{V}_1\right\|_F^2}}{\kappa} \tag{21}$$

where $(.)^{-1}$ denotes the inverse of a matrix

To obtain the matrices $\mathbf{D}_1$, $\mathbf{D}_2$ in practice, the relation defined in Eq. (10) is rewritten as follow:

$$\widetilde{\mathbf{A}} = \mathbf{A} + \delta\mathbf{A}\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\left(\mathbf{I}_{n_y} + \delta\mathbf{A}\mathbf{A}^{-1}\right) \quad (22)$$

By identifying Eq. (22) to Eq. (9), we obtain:

$$\mathbf{D}_1 = \mathbf{I}_{n_y}, \mathbf{D}_2 = \mathbf{I}_{n_y} + \delta\mathbf{A}\mathbf{A}^{-1} \quad (23)$$

The bound β_{M} evaluates the amount that a disturbance away *nonlinearly* (ANL) a monitored structure from an initial behavior to a new one.

EXPERIMETAL SETUP

The test bench used in this study was realized in [6, 7] and the presented section is based on the experimental data obtained from these studies. Figure 1.a shows the experimental setup of an aluminum plate $(14 \times 12 \times 0.078\,\mathrm{mm}^3)$, loaded in an uniaxial testing machine. The testing machine is equipped with an environmental chamber to collect sensor signals under varying temperature conditions. In figure 1.b, the schematic of the coupon is presented. It is surface mounted with a 2×3 of piezoelectric transducers, actuated using standard tone-burst signal (5-cycle Gaussian with Hanning window) at center frequency of $250\mathrm{kHZ}$, and sensed using a 64-channel data acquisition system provided by Acellent Technologies Inc. Sampling frequency for data collection was set at $12\mathrm{MHZ}$, and the signals were collected in pitch-catch mode, wherein one transducer acts as an actuator while the others act as sensors. Through that mode, a total of 9 sensors were recorded $n_y = 9$, with samples number $N = 1400$.

Sensor data of the healthy state were collected under different loading: from 0.0 kips to 7.0 kips and varying temperature conditions: from $22°\mathrm{C}$ to $70°\mathrm{C}$. Once the healthy state data collected, accelerated fatiguing of the coupon was carried out resulting in a crack of size of $5\mathrm{mm}$, located at a preexisting notch at the center of the coupon (Figure 1. b). For this damaged state, different loading: from 0.0 kips to 7.0 kips and varying temperature conditions: from $22°\mathrm{C}$ to $70°\mathrm{C}$ were realized.

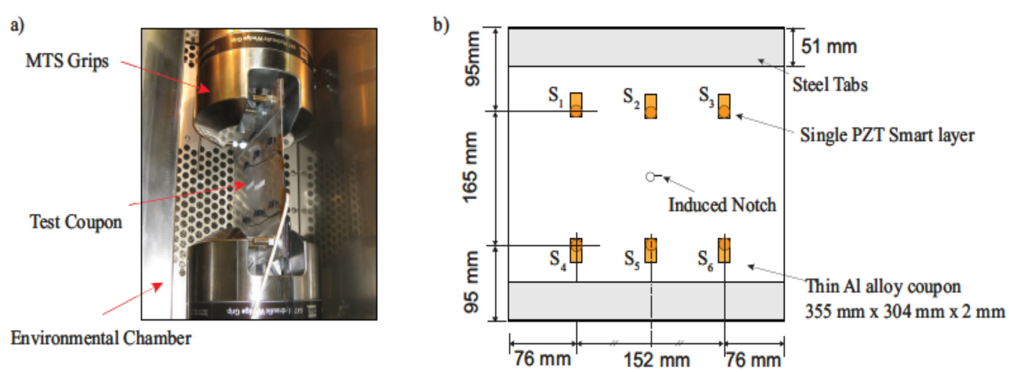


Figure 1: a) Experimental set up of a test coupon loaded in the MTS uniaxial testing machine with environmental chamber, and b) schematic of the same test coupon with attached array of piezoelectric transducers; Details on the experimental set up in [6, 7].

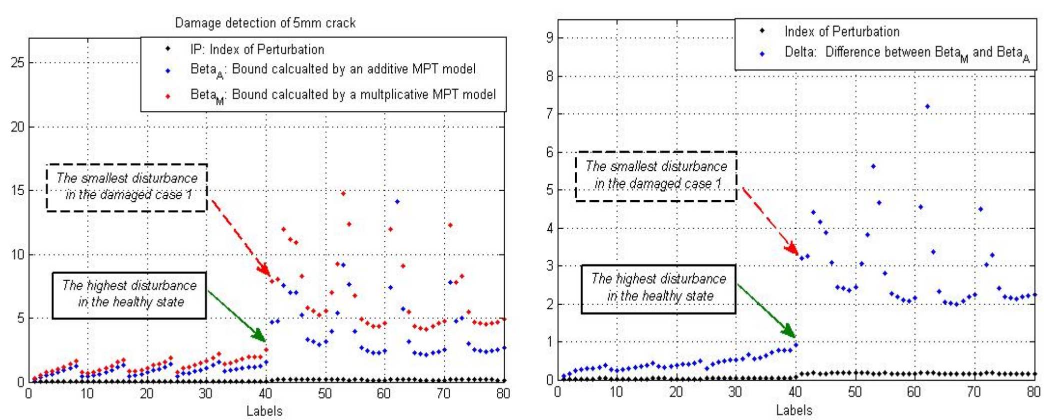
TEST RESULTS

In this section, the bases of matrix perturbation theory are interpreted through the presented experimental test bench. It is to be noted that the data used in our work were obtained in [6, 7] for purpose of damage detection methodology with a reference database. To exploit that database in our concern, we have set the initial behavior of the monitored structure (noted R) as the healthy state condition corresponding to: *unloaded and temperature 22°C*. Accordingly, matrix $\mathbf{A}$ is that obtained from BSS technique on the data of R, matrix $\widetilde{\mathbf{A}}$ reflects the new behavior of the monitored structure, and it is calculated for each tested data. Figure 2.a illustrates the results of the described quantities: index of perturbation IP, the bounds β_{A} and β_{M} defined respectively in Eqs. (7), (18) and (21), an those for different healthy state conditions and damage case 1: crack of 5mm.

For each label, there is a well-defined value for IP, β_{A} and β_{M}. On a first finding, one can see that the IP is higher to the presence of damage case of 5mm crack than the disturbance occurred in the healthy state conditions. Indeed, the IP calculated for the *highest* studied disturbance occurred in the healthy state: *load of 7 $kips$, temperature* $70°C$ is less than that obtained from the *lowest* disturbance occurred for damage case 1, *i.e. loads of 0.0, temperature* 30°C.

Now, the analytical bounds β_M and β_A come to play. The first one is based on the multiplicative perturbation model defined in Eq. (9), and it quantifies the amount of how much a disturbance away nonlinearly a monitored structure from an initial behavior to a new one. Referring still to figure 2.a, one can see that for labels 41 to 80, there is a significant disparity between IP and β_M: the nonlinearly trend is sharper for theses labels. Hence, the occurred disturbance is suspected to be a damaged state.

To confirm that, the gap between β_M and β_A is evaluated by calculating the quantity $\delta = \beta_M - \beta_A$, and it is illustrated in figure 2.b. One can see that there is a significant gap for labels 41 to 80: disturbance attracts the monitored structure to a nonlinearity trend, *i.e.* the structure is in damaged state. While there is a small gap for labels 1 to 40: the monitored structure is in a healthy state, subjected to environmental/operational conditions.



(a) (b)

Figure 2: Comparison between the healthy state and damage case of 5mm crack:

(a) Evolution of the index of perturbation & additive/multiplicative bounds through the studied cases

(b) Evolution of the index of perturbation and the gap between the bounds for each studied cases

Labels used in figure 2

- From *i*=1 to 40: label *i* is defined by comparing the reference R versus the tested data of the healthy state H_i 0.0 kips to 7.0 kips and varying temperature conditions: from 22°C to 70°C
- From *i*=41 to 80: label i is defined by comparing the reference R versus the tested data of the damaged case 1 (5mm)

CONCLUSION

This work has dealt an essay for the baseline free damage detection. Specifically, we have proposed two antagonists concepts that assume respectively that unknown disturbance can away linearly (AL) and nonlinearly (ANL) a monitored structure from an initial behavior to a new one. The introduced concepts have been demonstrated through an experimental setup of a test coupon aluminum plate, which has undergone varying loads and temperatures conditions, and crack damage.

The results described here are initial attempts for a complete online SHM technique. For the work under progress, we are investigating a methodology to track the crack damage cases.

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