Title: A Sequential Statistical Time Series Framework for Vibration Based Structural Health Monitoring

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ABSTRACT

The goal of this study is the introduction and experimental assessment of a Sequential Probability Ratio Test (SPRT) framework for vibration based Structural Health Monitoring (SHM). This employs the residual sequences obtained using a single stochastic time series model of the healthy structure and is based on a combination of binary and multihypothesis versions of the SPRT. The framework’s performance is predetermined via the use of the Operating Characteristic (OC) and Average Sample Number (ASN) functions in combination with baseline experiments, while it requires on average a minimum number of samples in order to reach a decision compared to Fixed Sample Size (FSS) most powerful tests. The effectiveness of the proposed approach is validated and experimentally assessed via its application to a lightweight aluminum truss structure.

INTRODUCTION

Statistical time series methods form an important, rapidly evolving class, within the broader vibration based family of Structural Health Monitoring (SHM) methods [1–4]. Their main elements are: (i) random excitation and/or vibration response signals (time series), (ii) statistical model building, and (iii) statistical decision making for inferring the health state of a structure. They offer a number of potential advantages, including no requirement for physics based or finite element models as they are data based (inverse type) methods, no requirement for complete modal models, effective treatment of uncertainties, and statistical decision making with specified performance characteristics [1, 2, 5].

The vast majority of statistical time series SHM methods is based on Fixed Sample Size (FSS) hypothesis testing used for the statistical decision making. On the other hand, sequential methods have the feature that the number of observations required is not determined in advance, but depends, at each stage, on the results of the observations previously made. Thus, the number of observations required by the test is not predetermined, but a random variable. A merit of the sequential approach is that test procedures

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can be constructed which require, on average, a substantially smaller number of observations than equally reliable test procedures based on a predetermined (fixed) number of observations and thus leading to an earlier decision [6]. Moreover, a potential advantage of a SHM method based on sequential procedures is its straightforward extension for online implementation, which is of high interest with respect to current SHM application requirements. In this context, preliminary – with respect to the use of a Sequential Probability Ratio Test (SPRT) scheme – studies include [7, 8], where the binary form of the SPRT based on AR-ARX model residuals has been applied for damage detection in a laboratory three-story building model and an eight-degree-of-freedom mass-spring system, respectively.

The goal of the present study is the introduction and experimental assessment of a model residual based sequential framework for SHM capable of achieving early and robust damage detection and identification (classification) under experimental uncertainties. This framework is based on the statistically optimal SPRT (both its binary and multi-hypothesis versions [9, 10]), while taking advantage – for the first time in the context of vibration based SHM – of its properties and capabilities. The basis of the proposed framework consists of the residual sequences obtained through a single stochastic time series model of the healthy structural dynamics.

The effectiveness of the framework is validated and experimentally assessed via its application to a lightweight aluminum truss structure. The results presented for three distinct vibration response measurement positions, with a single measurement used at a time, confirm its ability to operate based even on a single pair of measured excitation–response signals. The damage cases correspond to the loosening of various bolts connecting certain of the truss elements. The main features and operational characteristics are discussed, while the effectiveness is demonstrated via various test cases corresponding to different experiments, damage types, and vibration measurement positions.

THE MODEL RESIDUAL BASED SEQUENTIAL FRAMEWORK

The postulated framework consists of two phases: (a) An initial baseline phase, which includes the modeling of the healthy structure, and (b) the inspection phase, which is performed during the structure’s service cycle or continuously (online), and includes the functions of damage detection and identification. A schematic representation of the sequential framework is presented in Figure 1.

Baseline Phase

Data records from the healthy structure are employed for the identification of an appropriate parametric time series model. Specifically a scalar (univariate) model is needed in case of a single vibration response measurement location, or a vector (multi-
The standard deviation, so that the structure is determined hea

inspection (ARX) models may be employed [11]. In the present study a single measurement

Inspection Phase

Damage detection and identification are based on the binary and multihypothesis versions of the SPRT, respectively [9, 10], which are used in order to detect a change in the standard deviation $\sigma$ of the model residual sequence obtained by driving the current (unknown) excitation $(x[t])$ and response $(y[t])$ signals through a single baseline healthy time series model. The SPRT allows for the specification of two values $\sigma_o$ and $\sigma_1$ for the standard deviation, so that the structure is determined healthy iff $\sigma \leq \sigma_o$, and damaged iff $\sigma \geq \sigma_1$. The zone between $\sigma_o$ and $\sigma_1$ constitutes an uncertainty zone, thus for $\sigma$ lying in it the decision is postponed and data collection continues. The values of $\sigma_o$ and $\sigma_1$ are user defined and express the increase of the standard deviation ratio $q = \sigma_1/\sigma_o$ for which the structure is considered to be damaged. For example, a ratio of $q = 1.1$ means that the structure is considered damaged whenever there is an increase of 10% in the standard deviation $\sigma$ of the current residual sequence compared to a threshold value $\sigma_o$.

Damage detection is based on the binary hypothesis testing problem implemented via the SPRT of strength $(\alpha, \beta)$, with $\alpha, \beta$ designating the type I (false alarm) and II (missed damage) error probabilities, respectively:

$$
H_0 : \sigma \leq \sigma_o \quad \text{(null hypothesis – healthy structure)}
$$

$$
H_1 : \sigma \geq \sigma_1 \quad \text{(alternative hypothesis – damaged structure)}
$$

with $\sigma$ designating the standard deviation of a scalar model residual signal $e[t]$ obtained by driving the current response signal through the healthy structural model, and $\sigma_o, \sigma_1$ user defined values. Under the null hypothesis of a healthy structure the residuals $e[t]$ are iid zero mean Gaussian with variance $\sigma^2$, hence $e[t] \sim \text{iid } \mathcal{N}(0, \sigma^2)$.

The basis of the SPRT is the logarithm of the likelihood ratio function which is computed at data sample $t$ (presently coinciding with discrete time) as follows:

$$
\Lambda[t] = \log \frac{f(e[1], \ldots, e[t]|H_1)}{f(e[1], \ldots, e[t]|H_0)} = \sum_{i=1}^{t} \log \frac{f(e[i]|H_1)}{f(e[i]|H_0)} = t \cdot \log \frac{\sigma_o}{\sigma_1} + \frac{\sigma_1 - \sigma_o}{2\sigma_o\sigma_1^2} \sum_{i=1}^{t} e^2[i], \quad t = 1, 2, \ldots
$$

with $\Lambda[t]$ designating the decision parameter of the method and $f(e[i]|H_i)$ the probability density function (normal distribution) of the residual sequence under hypothesis $H_i$ ($i = 0, 1$).

Decision making is then based on the test (of strength $(\alpha, \beta)$):

$$
\Lambda[t] \leq \log B \quad \text{accept } H_0 \quad \text{(healthy structure)}
$$

$$
\Lambda[t] \geq \log A \quad \text{accept } H_1 \quad \text{(damaged structure)}
$$

$$
\log B < \Lambda[t] \leq \log A \quad \text{no decision is made} \quad \text{(continue the test)}
$$

with $A = (1 - \beta)/\alpha$ and $B = \beta/(1 - \alpha)$. Following a decision at a stopping sample (time) $\hat{T}$, it is possible to continue the test by resetting $\Lambda[\hat{T} + 1]$ to zero and continuing by collecting additional residual samples.
For any value of the residual standard deviation $\sigma$, the Operating Characteristic (OC) function of the SPRT denotes the probability that the test will terminate with the acceptance of the null hypothesis $H_0$ [9]. Similarly, the Average Sample Number (ASN) function represents the average number of inspection samples required by the SPRT to reach a decision [9]. The ASN is an approximation of the expected value $E(o|T)$ of the number of residual samples required by a sampling plan of strength $(\alpha, \beta)$ and standard deviations $\sigma_0, \sigma_1$ in order to reach a terminal decision.

Damage identification is based on the multi-hypothesis sequential test, which is based on the Armitage test [9,10]. Then, considering $k$ hypotheses ($k$ potential damage states), the multi-hypothesis test to be implemented may be expressed as follows:

$$H_A : \sigma = \sigma_A \quad \text{Hypothesis } A - \text{damage is of type } A$$

$$H_B : \sigma = \sigma_B \quad \text{Hypothesis } B - \text{damage is of type } B$$

$$\vdots \quad \vdots \quad \vdots$$

The standard deviation values $\sigma_A, \sigma_B, \ldots$ are user defined and may be determined based on available baseline data obtained from the structure under damage types $A, B, \ldots$, respectively. A typical selection of $\sigma_A, \sigma_B, \ldots$ could be as the mean values of the residual standard deviations estimated from the available baseline data records under the corresponding damage structural states. By denoting the log likelihood under hypothesis $H_i$ ($H_i$ is true, $i = A, B, \ldots$) as $L_i$ there are $\frac{k(k - 1)}{2} \log$ likelihood ratios for the various pairs of hypotheses, with each one expressed in terms of $k - 1$ independent likelihood ratios [10,12]:

$$\Lambda_{ij}[t] = \frac{L_i[t]}{L_j[t]} = t \cdot \log \frac{\sigma_j}{\sigma_i} + \frac{\sigma_i^2 - \sigma_j^2}{2\sigma_i^2} \cdot \sum_{t=1}^{T} \epsilon^2[t] \quad i, j = A, B, \ldots \text{ and } i \neq j. \quad (5)$$

Then, the multi-hypothesis test termination is defined by the pair $(T, \delta)$, with $T$ indicating the stopping time and $\delta$ the final decision [12, pp. 237–238]:

$$\tilde{T} = \min_j \inf \left\{ t : \Lambda_{ij}[t] \geq \log A_{ij} \forall i \neq j, i < j, t = 1, 2, \ldots \right\}, \quad \tilde{\delta} = \arg \min_{j=1,\ldots,k} T. \quad (6)$$

Let $a_{ij}$ the probability of accepting $H_i$ when in fact $H_j$ is true (error probabilities), that is $\alpha_{ij} = P(\delta = H_i/H_j), \ i \neq j$, and let $a_{ii}$ the probability of accepting $H_i$ when in fact $H_i$ is true (correct decision probabilities), that is $\alpha_{ii} = P(\delta = H_i/H_i)$. The error probabilities $a_{ij}$ may be controlled via suitable selection of the $A_{ij}$’s [9,10].

It is possible that different damage types may have a similar effect on the residual sequences and thus in the residual standard deviation. Thus, the multi-hypothesis method will not provide clear classification results for the corresponding damage types, but an indication of the potential types. In this case the user may apply, in a second stage, the binary SPRT between the damage types indicated by the multi-hypothesis testing.

**THE STRUCTURE AND THE EXPERIMENTAL SET-UP**

The truss structure is suspended through a set of cords and consists of twenty eight elements with rectangular cross sections ($15 \times 15$ mm) jointed together via steel elbow plates and bolts (Figure 2). All parts are constructed from standard aluminum with the
overall dimensions being $1400 \times 700 \times 800$ mm. The force excitation is a random Gaussian signal applied vertically at Point X via an electromechanical shaker (MB Dynamics Modal 50A, max load 225 N) equipped with a stinger, and measured via an impedance head (PCB 288D01, sensitivity 98.41 mV/lb). The vibration responses are measured at different points via dynamic strain gauges (PCB ICP 740B02, 0.005 – 100 kHz, 50 mV/µε; sampling frequency $f_s = 256$ Hz, signal bandwidth 0.5 – 100 Hz). The force and strain signals are driven through a signal conditioning device (PCB 481A02) into the data acquisition system (SigLab 20–42). In this study the damage detection and identification results are obtained based on each one of three vibration response signals (Points Y1, Y2 and Y3 – Figure 2). This allows the examination and assessment of the proposed framework’s ability to achieve damage detection and identification with respect to the vibration response measurement positions employed. For this reason, damage is characterized as “local” or “remote” with respect to the sensor used.

1200 and 900 experiments for the healthy and damaged structural states, respectively, are undertaken, 100 of which are employed in the baseline phase – the rest are used in the inspection phase; see Table I). In each experiment vibration measurements are collected at Points Y1, Y2, Y3 (Figure 2). Further experimental details are provided in Table I – worth noting is the very low/limited bandwidth used. The sample mean is subtracted from each signal and scaling by the signal’s sample standard deviation is implemented.

**Figure 2.** The aluminum truss structure and the experimental set-up: The force excitation (Point X), the vibration measurement positions (Points Y1–Y3), and the considered damage types.

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Description</th>
<th>Total Number of Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td></td>
<td>1200 (100 baseline)</td>
</tr>
<tr>
<td>Damage type A</td>
<td>loosening of bolt A1</td>
<td>900 (100 baseline)</td>
</tr>
<tr>
<td>Damage type B</td>
<td>loosening of bolts A1 and B1</td>
<td>900 (100 baseline)</td>
</tr>
<tr>
<td>Damage type C</td>
<td>loosening of bolts C1 and C2</td>
<td>900 (100 baseline)</td>
</tr>
<tr>
<td>Damage type D</td>
<td>loosening of bolt D1</td>
<td>900 (100 baseline)</td>
</tr>
<tr>
<td>Damage type E</td>
<td>loosening of bolt E1</td>
<td>900 (100 baseline)</td>
</tr>
</tbody>
</table>

Sampling frequency: $f_s = 256$ Hz, Signal bandwidth: [0.5 – 100] Hz
Signal length $N$ in samples (s): Non-parametric analysis: $N = 30720$ (120 s) Parametric analysis: $N = 1000$ (3.9 s)
DAMAGE DETECTION AND IDENTIFICATION RESULTS

Baseline Phase: Structural Identification Under the Healthy Structural State

Parametric identification of the structural dynamics is based on \( N = 10\,000 \approx 39 \) s) sample-long excitation and single response signals which are used for estimating AutoRegressive with eXogenous excitation (ARX) models (MATLAB function `arx.m`). The modeling strategy consists of the successive fitting of ARX(\( na, nb \)) models (with \( na, nb \) designating the AR and X orders, respectively; in this study \( na = nb = n \)) until a suitable model is selected. Model parameter estimation is achieved by minimizing a quadratic Prediction Error (PE) criterion leading to a Least Squares (LS) estimator [11, p. 206]. Model order selection is based on the BIC and RSS/SSS (Residual Sum of Squares / Signal Sum of Squares) criteria, and the use of frequency stabilization diagrams [11]. This procedure leads to the selection of an ARX(112, 112), ARX(136, 136) and ARX(103, 103) model for vibration measurement positions Y1, Y2 and Y3, respectively. The selected models are summarized in Table II. Note that the identification procedure generally leads to different ARX models (including somewhat different model orders) for each vibration measurement position.

Inspection Phase

Prior to implementing the SPRT, an appropriate sampling plan should be selected. The selection of the sampling plan involves the determination of the following three aspects: (i) the nominal residual standard deviation \( \sigma_o \) under which the structure is considered to be in its healthy state, (ii) the standard deviation ratio \( q = \sigma_1 / \sigma_o \), which constitutes the standard deviation increase under which the structure is determined to be damaged, and (iii) the SPRT strength \((\alpha, \beta)\).

The determination of the residual standard deviation \( \sigma_o \) under which the structure is considered healthy is based on the available 100 baseline data records obtained from the healthy structure (Table I). The value \( \sigma_o \) is chosen in order for the probability of \( \sigma \leq \sigma_o \) to be equal to 95\% \( (P(\sigma \leq \sigma_o) = 0.95) \). The determination of the residual standard deviation ratio \( q \) may be based on the OC and ASN functions of the SPRT [9] for various \( q \) ratios, along with the use of the baseline data records. Figures 3a and 3b present, for vibration response Y1, the OC and ASN functions, respectively, for various candidate ratios \( q \) and constant SPRT strength \((\alpha, \beta) = (0.01, 0.01)\). In both figures, the \( \sigma_o \) value is shown as gray vertical dashed line, while the \( \sigma_1 \) values corresponding to the considered \( q = \sigma_1 / \sigma_o \) ratios are shown in colored vertical dashed lines. Along with the OC and ASN function curves, the standard deviation values obtained from the 100 baseline residual sequences are depicted in vertical cyan dashed lines.

In Figure 3a the intersections of the dashed vertical lines, belonging to the residual standard deviation values, with the OC function curves for the various \( q \) ratios corre-

<table>
<thead>
<tr>
<th>TABLE II. SELECTED MODELS AND ESTIMATION DETAILS.</th>
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<tbody>
<tr>
<td>Response</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>Y1</td>
</tr>
<tr>
<td>Y2</td>
</tr>
<tr>
<td>Y3</td>
</tr>
</tbody>
</table>
Figure 3. Healthy structure: (a) Operating Characteristic (OC) and (b) Average Sample Number (ASN) functions for various residual standard deviation ratios $q = \sigma_1 / \sigma_o$ and constant strength $(\alpha, \beta) = 0.01$. The vertical colored dashed lines designate the $\sigma_1$ values for the corresponding ratios $q$. The dashed vertical cyan lines represent the residual standard deviation values for each of the 100 baseline healthy data sets.

spond to the probabilities of acceptance of the null hypothesis $H_o$ (healthy structure) for each ratio, while in Figure 3b correspond to the expected number of residual samples required to reach a decision. The OC function (Figure 3a) is considered more favorable the higher the value of $L(\sigma)$ for $\sigma$ consistent with $H_o$ and the lower the value of $L(\sigma)$ for $\sigma$ not consistent with $H_o$. Thus, by plotting the OC and ASN functions, not only one may have an indication of the probability of acceptance for various residual standard deviations $\sigma$, but one may also obtain an approximation of the number of residual samples that are required for reaching a terminal decision.

Indicative damage detection results for Point Y3 are presented in Figure 4. A damage is detected when the test statistic (vertical axis) exceeds the upper critical point (dashed horizontal lines), while the structure is determined as being in its healthy state when the test statistic exceeds the lower critical point. After a critical point is exceeded a decision is made, while the test statistic is reset to zero and the test continues. Hence, during testing multiple decisions may be made. Evidently, correct detection is obtained in each test case, as the test statistic is shown to exceed multiple times (multiple correct decisions) the lower critical point in the healthy case, while it also exceeds multiple times the upper critical point (multiple correct damage detections) in the damage test cases.

The summarized damage detection results are presented in Table III. The false alarm rates are extremely low, as well as the mean missed damage rates which are zero, except for damage type A which exhibits a somewhat increased number of missed damage rate when the response Y1 is used. Summary identification results for all vibration responses are presented in Table IV. The correct damage classification percentages are presented for all damage type inspection sets, along with the corresponding mean stopping times. As it may be observed the multihypothesis test damage classification results obtained for all vibration responses are very accurate for damage types A, C and E, as the percentages of correct classification are very high. Nevertheless, the method faces difficulties in accurately classifying damage types B and D. As already mentioned, this is due to the fact that these damage types have a similar effect on their corresponding residual standard
Figure 4. Indicative damage detection results (response Y3) at the $(\alpha, \beta) = (0.01, 0.01)$ risk levels ($q = \sigma_1/\sigma_o = 1.1$). The actual structural state is shown above each plot.

deviation values obtained through the healthy models. In this case, the user may apply the binary SPRT for the candidate damage types. Nevertheless, this procedure would require the baseline modeling of at least one of these types.

CONCLUDING REMARKS

A model residual based sequential framework for SHM was introduced. Damage detection and identification were effectively tackled, achieving high performance with practically zero false alarms and missed damage rates. An optimal sampling plan was determined a priori via the use of the Operating Characteristic (OC) and Average Sample Number (ASN) functions, selected type I (false alarm) and II (missed damage) error probabilities, and available baseline data records under various potential states. Early (needing at maximum 0.9 s) and robust damage detection were achieved, and “local” and “remote” damage with respect to the sensor position was detected. The multihypothesis

<table>
<thead>
<tr>
<th>TABLE III. DAMAGE DETECTION SUMMARY RESULTS.</th>
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<tbody>
<tr>
<td><strong>Response</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Y1</td>
</tr>
<tr>
<td>Y2</td>
</tr>
<tr>
<td>Y3</td>
</tr>
</tbody>
</table>

Test strength $(\alpha, \beta) = 0.01$; Residual standard deviation ratio $q = \sigma_1/\sigma_o = 1.1$.
**Mean healthy detections** and **false alarms** per data set out of 1100 healthy inspection experiments.
**Mean missed damage values** per data set out of 900 damage inspection experiments.
TABLE IV. DAMAGE IDENTIFICATION SUMMARY RESULTS.

<table>
<thead>
<tr>
<th>Actual damage</th>
<th>Damage classification (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>damage A</td>
</tr>
<tr>
<td></td>
<td>hypothesis</td>
</tr>
<tr>
<td>Type A</td>
<td>99.33/98.22/100</td>
</tr>
<tr>
<td>Type B</td>
<td>0/0/0</td>
</tr>
<tr>
<td>Type C</td>
<td>0/0/0</td>
</tr>
<tr>
<td>Type D</td>
<td>0/0/0</td>
</tr>
<tr>
<td>Type E</td>
<td>0/0/0</td>
</tr>
</tbody>
</table>

Mean stop. time: 15.68/22.84/8.76, 174.35/172.32/176.26, 36.18/117.99/3.81, 200.08/167.34/231.98, 18.54/90.33/27.21

Damage classification: percentage for points Y1/Y2/Y3 out of 800 inspection experiments; $\alpha_{ij} = 0.01$.

Mean stopping time: in samples for points Y1/Y2/Y3 out of 800 inspection experiments of 1000 samples.

The test based damage identification procedure faced some difficulties in classifying one damage type, an issue that may be tackled via the baseline modeling of the specific damage type followed by binary SPRT testing.

REFERENCES


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