Vibration Based Health Monitoring for a Thin Aluminum Plate: Experimental Assessment of Several Statistical Time Series Methods

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ABSTRACT

This work aims at the experimental assessment of a number of statistical time series methods for Structural Health Monitoring (SHM). The main features and operation of the employed non–parametric and parametric methods are briefly reviewed. Their performance is subsequently assessed via laboratory experiments pertaining to damage detection and identification on a thin aluminum plate structure. The results of the study demonstrate the potential and effectiveness of the statistical time series SHM methods.

INTRODUCTION

Vibration based statistical time series methods for Structural Health Monitoring utilize random excitation and/or vibration response signals, along with statistical model building and decision making tools, for inferring the health state of a structure [1–5]. They offer a number of important advantages, including no requirement for physics–based or finite element type models, no requirement for complete modal models, the treatment of uncertainties, and statistical decision making with specified performance characteristics. In spite of these, the literature on vibration–based time series methods for condition monitoring remains relatively sparse, and, in particular, no application studies that assess and experimentally compare the various methods are available.

The *goal* of the present study is to present the application of a number of statistical time series methods to damage detection and identification in a thin aluminum

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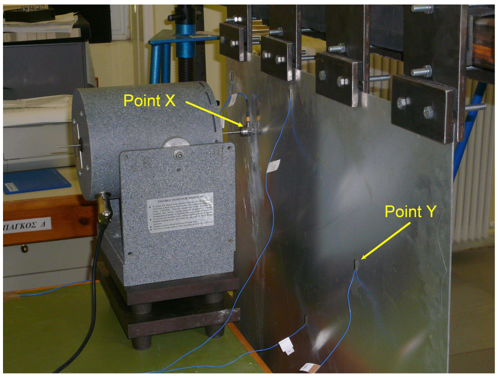

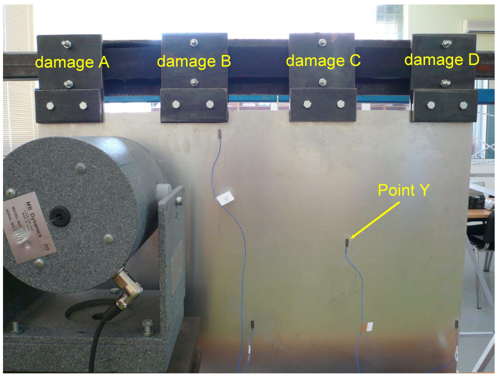

Figure 1. The thin plate structure and the experimental set-up: The force excitation position (point X), the vibration measurement position (Point Y), and the considered damage types.

plate. The methods are based on a single pair of (scalar) excitation–response vibration signals obtained via force and dynamic strain gauge sensors, respectively. Four types of structural conditions are considered, each one corresponding to the release of each one of the four clamps (clamps A, B, C, D) holding the plate (Figure 1). Two non-parametric methods, namely a Power Spectral Density (PSD) and Frequency Response Function (FRF) based method, and three parametric methods, namely a model parameter based, a residual variance, and a residual uncorrelatedness based method, are employed and are experimentally assessed via a number of test cases.

THE EXPERIMENTAL SET-UP

The structure. The thin plate structure is depicted in Figure 1. It is made of standard aluminum, with its dimensions being $1000 \times 800 \times 2$ mm, and it is suspended vertically via four symmetrically positioned clamps designated as clamps A, B, C, and D. Each clamp consists of two thick (5 mm) rectangular steel plates tightened together via two M8 bolts.

The damage and the experiments. Four distinct damage types are considered (Table I). Each one, referred to as *Damage type A, B, C, or D*, corresponds to the complete loosening of clamp A, B, C, or D, respectively (Figure 1).

In all cases the excitation is random Gaussian force applied horizontally at Point X (Figure 1) via an electromechanical shaker (MB Dynamics Modal 50A) equipped

TABLE I. STRUCTURAL STATES AND EXPERIMENTAL DETAILS

Structural State	Description	Number of Experiments
Healthy	—	13
Damage type A	loosening of clamp A	5
Damage type B	loosening of clamp B	5
Damage type C	loosening of clamp C	5
Damage type D	loosening of clamp D	5
Sampling frequency: $f_s = 256$ Hz; signal bandwidth: $[0.5 - 100]$ Hz		
Signal length N in samples (s):		
Non–parametric methods: $N = 30\ 720$ (120 s); Parametric methods: $N = 6\ 480$ (25 s)		

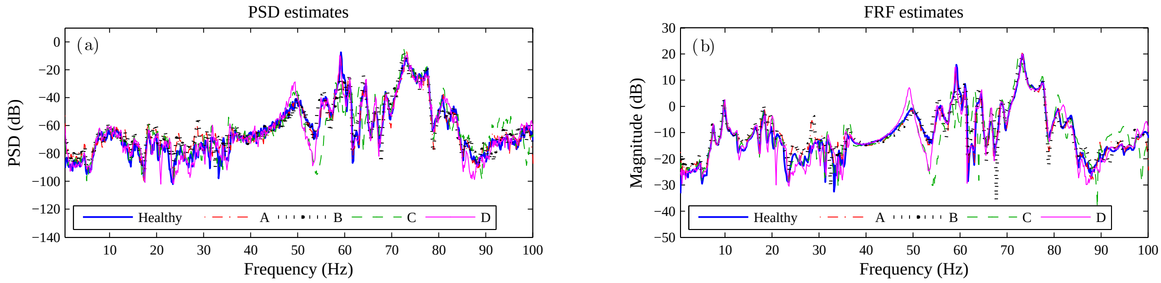

Figure 2. (a) Power Spectral Density (PSD) and (b) Frequency Response Function (FRF) magnitude estimates for the healthy and damaged structural states (Welch method – details in Table III).

with a stinger. The actual force exerted on the structure is measured via an impedance head (PCB 288D01), while the resulting vibration responses are measured (sampling frequency $f_s = 256$ Hz, signal bandwidth $0.5 - 100$ Hz) at various points on the structure via strain gauges (PCB ICP 740B02). In this study the results are based on a *single* vibration response measurement (Point Y, Figure 1) are presented. The force and strain signals are driven through a signal conditioning device (PCB 481A02) into the data acquisition system (SigLab 20–42).

A number of experiments are carried out, initially for the healthy structure (13 experiments) and subsequently for each damaged state (5 experiments per state – see Table I). Each measured signal is adjusted to zero sample mean and normalized to a unity sample standard deviation. Typical non–parametric (Welch based) Power Spectral Density (PSD) and Frequency Response Function (FRF) magnitude estimates for the healthy and each damaged state of the structure are depicted in Figure 2.

STATISTICAL TIME SERIES METHODS FOR SHM

In this section the basic principles of the employed methods are presented. Each method is based on the detection of changes in a characteristic quantity Q constructed from each data set under the healthy/nominal and damaged structural states. Depending on the way this characteristic quantity is constructed, the methods are classified as non–parametric or parametric. The main characteristics of the methods are summarized in Table II. For a detailed overview the reader is referred to references [1, 2].

Non–parametric methods. Non–parametric methods are those in which the characteristic quantity Q is constructed based on non–parametric time series representations (models). Two non–parametric methods are used in this study: a Power Spectral Density (PSD) and a Frequency Response Function (FRF) based method. For both methods the excitation and/or vibration response signals are segmented into K non–overlapping sections, each of length L samples, and Welch type spectral estimation is employed.

(a) The Power Spectral Density (PSD) based method. This method tackles damage detection and identification via changes in the PSD $S(\omega)$ of the measured vibration signal when the excitation is not available (response–only case). The method's char-

TABLE II. STATISTICAL TIME SERIES METHODS FOR SHM [2]

Method	Null Hypothesis (principle)	Test Statistic
PSD	$S_u(\omega) \stackrel{?}{=} S_o(\omega)$	$F = \widehat{S}_o(\omega)/\widehat{S}_u(\omega)$
FRF	$\delta\lvert H(j\omega)\rvert = \lvert H_o(j\omega)\rvert - \lvert H_u(j\omega)\rvert \stackrel{?}{=} 0$	$Z = \delta\lvert \widehat{H}(j\omega)\rvert/\sqrt{2}\widehat{\sigma}_H$
Model parameter	$\delta\boldsymbol{\theta} = \boldsymbol{\theta}_o - \boldsymbol{\theta}_u \stackrel{?}{=} \mathbf{0}$	$\chi^2_\theta = \delta\widehat{\boldsymbol{\theta}}^T(2\widehat{\boldsymbol{P}}_\theta)^{-1}\delta\widehat{\boldsymbol{\theta}}$
Residual variance	$\sigma^2_{oo} \stackrel{?}{\geq} \sigma^2_{ou}$	$F = \widehat{\sigma}^2_{ou}/\widehat{\sigma}^2_{oo}$
Residual likelihood	$\boldsymbol{\theta}_o \stackrel{?}{=} \boldsymbol{\theta}_u$	$\chi^2_N = N\widehat{\sigma}^2_{ou}/\sigma^2_{oo}$
Residual uncorr.	$\rho[\tau] \stackrel{?}{=} 0$	$\chi^2_\rho = N(N+2)\sum_{\tau=1}^{r}(N-\tau)^{-1}\hat{\rho}^2[\tau]$

$S(\omega)$: Power Spectral Density (PSD) function; $\lvert H(j\omega)\rvert$: Frequency Response Function (FRF) magnitude
σ_H: standard deviation of $\lvert\widehat{H}_o(j\omega)\rvert$; $\boldsymbol{\theta}$: model parameter vector; $\boldsymbol{P}_\theta$: covariance of $\boldsymbol{\theta}_o$
σ^2_{oo}: variance of residual signal obtained by driving the healthy structure signals through the healthy model
σ^2_{ou}: variance of residual signal obtained by driving the current structure signals through the healthy model
$\rho[\tau]$: residual normalized autocovariance; N: signal length in samples
Estimators/estimates are designated by a hat.
The subscripts "o" and "u" designate healthy and current (unknown) structural state, respectively.

acteristic quantity thus is $Q = S(\omega)$. Damage detection is based on confirmation of statistically significant deviations (from the nominal/healthy) in the current structure's PSD function at some frequency(ies) [1, 2]. Damage identification may be achieved via a hypothesis testing procedure for each potential damage type.
(b) The Frequency Response Function (FRF) based method. This method is similar, but requires the availability of both the excitation and response signals and uses the FRF magnitude as its characteristic quantity, thus $Q = \lvert H(j\omega)\rvert$ with j designating the imaginary unit. Damage detection is based on confirmation of statistically significant deviations (from the nominal/healthy) in the current structure's FRF at some frequency(ies) through a hypothesis testing problem (for each ω) [1, 2]. Damage identification may be achieved via a hypotheses testing procedure for each potential damage type.

Parametric methods. Parametric methods are those in which the characteristic quantity Q is constructed based on parametric time series representations (models). They are applicable to both the response–only and excitation–response cases, as each situation may be dealt with through the use of proper representations. Two families of parametric methods are used in the study.
(a) The model parameter based method. This method attempts damage detection and identification using a characteristic quantity Q that is function of the parameter vector $\boldsymbol{\theta}$ of a time series model. The model has to be re–estimated during the inspection phase based on signals from the current (unknown) state of the structure. Damage detection is based on testing for statistically significant changes in the parameter vector $\boldsymbol{\theta}$ between the nominal and current structural states via a hypothesis testing procedure [1, 2]. Damage identification may be achieved via a hypotheses testing procedure for each potential damage type.
(b) Model residual based methods. These attempt damage detection and identification using characteristic quantities that are functions of residual sequences obtained by driving the current structural excitation and/or response signals through suitable

TABLE III. NON–PARAMETRIC METHOD DETAILS

Method	Segment length (L)	Non-overlapping segments (K)	Window type
Welch	2048 samples	15	Hamming

TABLE IV. PARAMETRIC METHOD DETAILS

Method	Estimated Model	Dimension of $\boldsymbol{\theta}$	Max. Lag r
Model parameter	ARX$(82, 82)$	165 parameters	–
Residual variance	ARX$(82, 82)$	165 parameters	–
Residual likelihood function	ARX$(82, 82)$	165 parameters	–
Residual uncorrelatedness	ARX$(96, 96)$	193 parameters	25 samples
ARX(na, nb) stands for AutoRegressive model with eXogenous excitation of orders (na, nb)			

pre–determined models corresponding to a particular state of the structure (healthy or damaged structure under specific damage type) [1, 2]. A first such method is based on the fact that the model matching the current state of the structure should generate a residual sequence characterized by minimal variance. A second method is based on the residual series uncorrelatedness. The model matching the current state of the structure should generate a white (uncorrelated) residual sequence. Both methods use classical tests on the residuals and offer simplicity and no need for model estimation in the inspection phase.

EXPERIMENTAL RESULTS

The presented methods are now applied to the problem of damage detection and identification on the thin aluminium plate. Non–parametric and parametric method details are provided in Tables III and IV, respectively.

Figures 3 and 4 present typical non–parametric damage detection results obtained via the PSD and FRF based methods, respectively. Evidently, correct detection at the selected α (false alarm) risk level is obtained in each case, as the test statistics are shown not to exceed the critical points (dashed horizontal lines) in the healthy case, while they clearly exceed them for each presented damage case. Note that damage type C appears easiest to detect, while damage type A appears as hardest (Figure 3). It is worth noting that damage A is close to the excitation point, but most distant from the response measurement point (Figure 1).

Figures 5 and 6 present typical parametric damage detection results obtained by the model parameter and residual uncorrelatedness based methods, respectively, at the selected α risk level. Evidently, correct detection is obtained in each case, as the test statistic is shown not to exceed the critical point in the healthy case, while it exceeds it in all considered damage cases (note the logarithmic scale on the vertical axis which indicates significant difference between the healthy and damage test statistics for the considered test cases).

Summary results for all the considered methods are presented in Table V. The damage detection assessment is based on 12 experiments for the healthy structure (an additional experiment is used for establishing the baseline) and 5 experiments for each considered damaged state (damage types A,. . . , D – see Table I). For damage

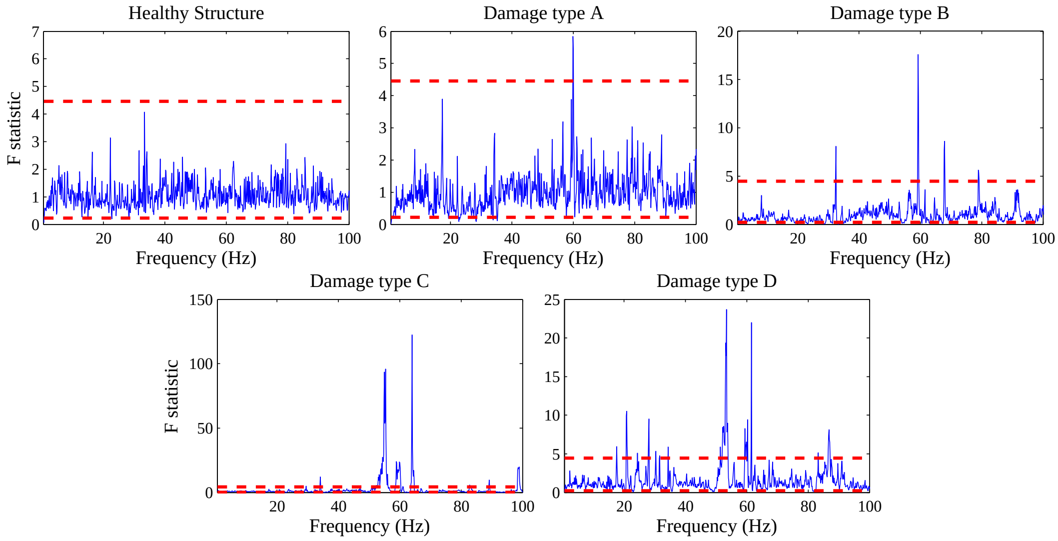

Figure 3. Indicative damage detection results via the PSD based method for five test cases (one healthy and four damaged) at the $\alpha = 10^{-4}$ risk level. A damage is detected if the test statistic exceeds the critical points (dashed horizontal lines).

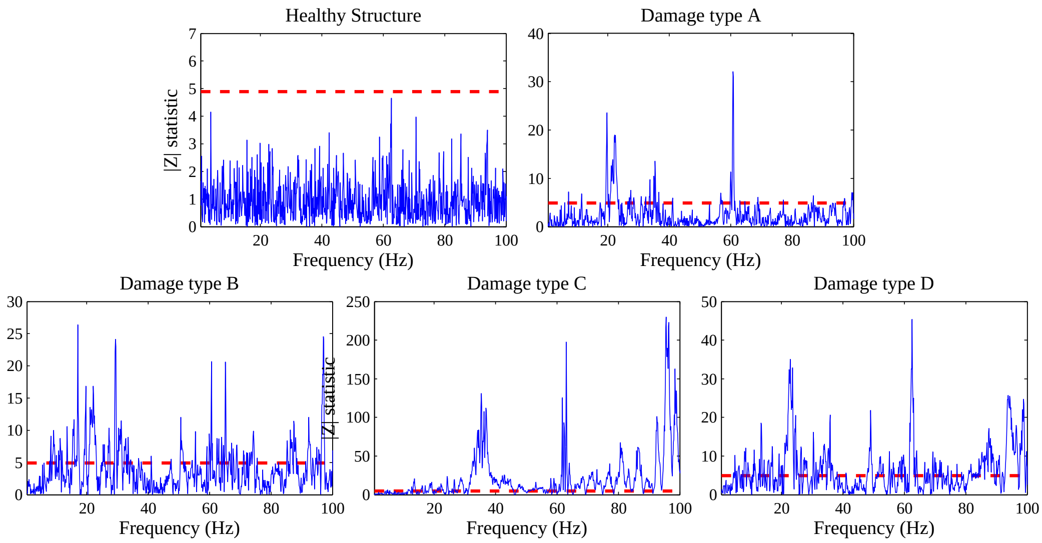

Figure 4. Indicative damage detection results via the FRF based method for five test cases (one healthy and four damaged) at the $\alpha = 10^{-6}$ risk level. A damage is detected if the test statistic exceeds the critical point (dashed horizontal line).

identification assessment one experiment (and corresponding data set) for each damage type is used for establishing the baseline, while the remaining 20 experiments compose the considered test cases.

As suggested by the results of Table V, both non–parametric and parametric methods achieve accurate damage detection with zero false alarms at the selected risk (false alarm) levels α and the vibration measurement position used. The ability of the methods to properly detect damage is accompanied by no missed damage cases, even for parametric methods for which a very small value of the risk level α was selected. Damage identification results also demonstrate the ability of the methods to accurately identify the actual damage type. It is worthwhile emphasizing that no damage

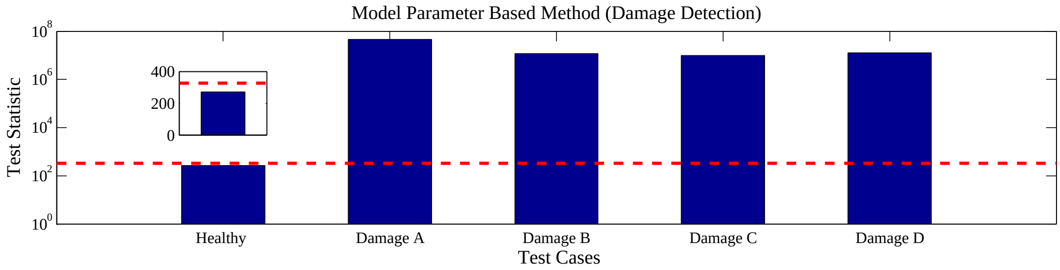

Figure 5. Indicative damage detection results via the model parameter based method (five test cases; critical point at the $\alpha = 10^{-12}$ risk level shown as dashed horizontal line; damage is detected if the test statistic exceeds the critical point).

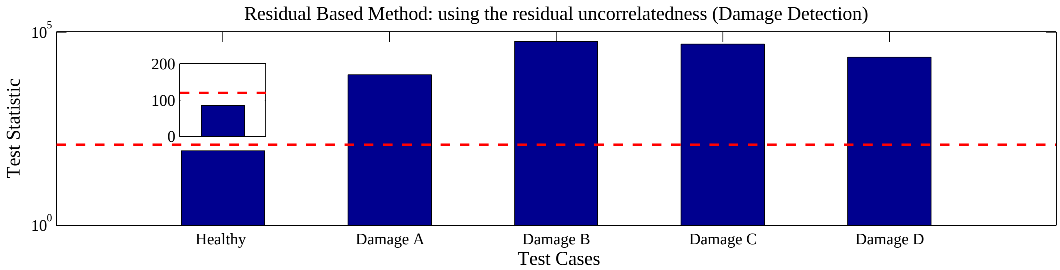

Figure 6. Indicative damage detection results via the residual uncorrelatedness based method (five test cases; critical point at the $\alpha = 10^{-12}$ risk level shown as dashed horizontal line; damage is detected if the test statistic exceeds the critical point).

misclassification errors are recorded.

Overall, both non–parametric and parametric time series methods for SHM demonstrate high potential for effective damage detection and identification, even when based on just a *single* vibration response signal. Furthermore, if the risk level α (false alarm) is properly adjusted, the methods seem to achieve accurate damage detection and damage type identification. The FRF and residual based methods seem to achieve clearer damage detection and damage type identification than the PSD and parameter based methods, respectively, although the performance of all considered methods appears very good.

Nevertheless, a number of issues require attention on part of the user. Effective model identification and proper selection of the risk level α (type I error) are crucial for successful damage diagnosis, especially for parametric methods. Moreover, In the case of multiple damage scenarios, statistical time series methods are capable of effectively treating damage detection, although proper damage identification (classification) is a more difficult problem that requires the use of advanced methods [4, 5].

CONCLUDING REMARKS

An experimental assessment of non–parametric and parametric statistical time series methods for SHM was presented via their application to damage detection and identification in a thin aluminum plate. Both types of methods were shown to effectively tackle the detection and identification subproblems, achieving excellent performance with zero false alarm, missed damage, and damage misclassification rates, although only a single vibration response signal measurement was used.

Non–parametric methods are generally simpler to use and require only little user

TABLE V. SUMMARY DAMAGE DETECTION AND IDENTIFICATION RESULTS

Method	Damage Detection					Damage Identification			
	False alarms	Missed damage				Damage misclassification			
		dam. A	dam. B	dam. C	dam. D	dam. A	dam. B	dam. C	dam. D
PSD based	0/12	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5
FRF based	0/12	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5
Mod. parameter†	0/12	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5
Res. variance†	0/12	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5
Res. likelihood†	0/12	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5
Res. uncor.†	0/12	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5

†adjusted α.

expertise. Parametric methods are somewhat more elaborate and require more experience. Yet, parametric methods offer increased sensitivity and accuracy, along with more effective tackling of the damage detection and identification subproblems. Accurate parametric modeling is nevertheless necessary, while the methods may be somewhat sensitive to experimental and modeling uncertainties.

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