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**SCALAR AND VECTOR TIME SERIES METHODS FOR VIBRATION BASED  
DAMAGE DIAGNOSIS IN AN AIRCRAFT SCALE SKELETON STRUCTURE**

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**ABSTRACT**

*A comparative assessment of several vibration based statistical time series methods for Structural Health Monitoring (SHM) is presented via their application to an aircraft scale skeleton structure. A concise overview of some of the main scalar and vector time series methods is provided, encompassing both non-parametric and parametric as well as response-only and excitation-response schemes. Damage detection and identification, collectively referred to as damage diagnosis, is based on single and multiple vibration response signals. The methods' effectiveness is assessed via multiple experiments under various damage scenarios (loosened bolts). The results of the study confirm the global damage detection capability and effectiveness of scalar and vector statistical time series methods for SHM.*

**INTRODUCTION**

Statistical time series methods for damage detection and identification (localization), collectively referred to as damage diagnosis, utilize random excitation and/or vibration response signals (time series), along with statistical model building and decision making tools, for inferring the health state of a structure (Structural Health Monitoring – SHM). They offer a number of advantages, including no requirement for physics based or finite

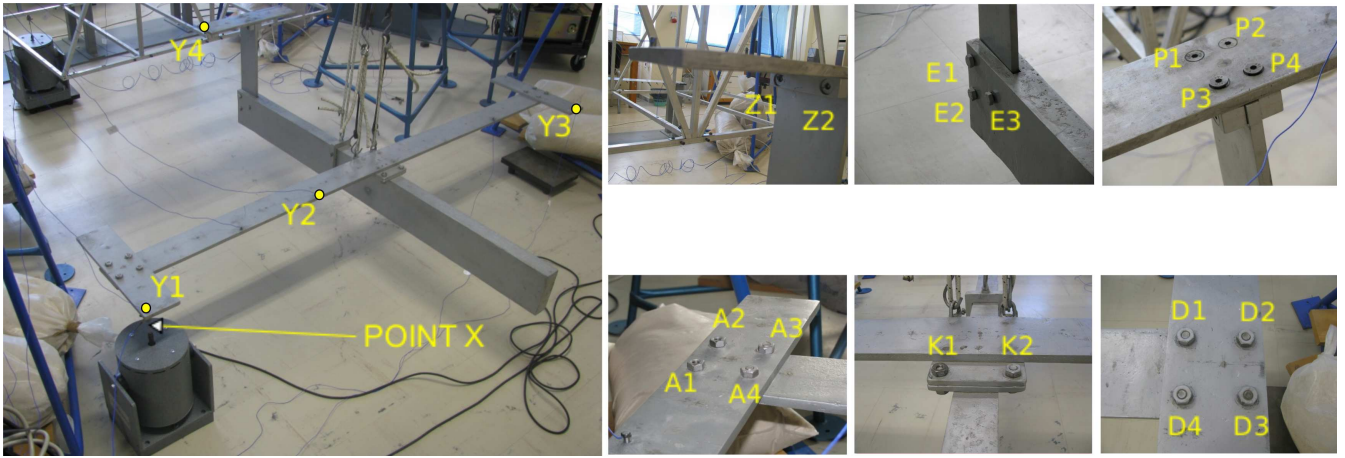
element models, no requirement for complete modal models, effective treatment of uncertainties, and statistical decision making with specified performance characteristics [1, 2]. These methods form an important, rapidly evolving, category within the broader vibration based family of methods [3–5].

Statistical time series methods for SHM are based on *scalar* or *vector* random (stochastic) vibration signals under healthy and potentially damaged states, identification of suitable (parametric or non-parametric) time series models describing the dynamics in each state, and extraction of a statistical characteristic quantity  $Q_o$  characterizing the structural state in each case (baseline phase). Damage diagnosis is then accomplished via statistical decision making consisting of comparing, in a statistical sense, the current characteristic quantity  $Q_u$  with that of each potential state as determined in the baseline phase (inspection phase). For an extended overview of the principles and techniques of statistical time series methods for SHM the interested reader is referred to the recent overviews by the last author and co-workers [1, 2].

Non-parametric time series methods are those based on scalar or vector non-parametric time series representations, such as spectral estimates [1, 2], and have received limited attention in the literature [6–8]. Parametric time series methods are those based on scalar or vector parametric time series representations, such as the AutoRegressive Moving Average (ARMA) models [1, 2]. This latter category has attracted significant attention re-

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**FIGURE 1.** THE AIRCRAFT SCALE SKELETON STRUCTURE AND THE EXPERIMENTAL SET-UP: THE FORCE EXCITATION (POINT X), THE VIBRATION MEASUREMENT LOCATIONS (POINTS Y1 – Y4), AND THE BOLTS CONNECTING THE VARIOUS ELEMENTS OF THE STRUCTURE.

cently [9–11].

The *goal* of the present study is the comparative assessment of several *scalar* (univariate) and *vector* (multivariate) statistical time series methods for SHM, both non-parametric and parametric, via their application to an aircraft scale skeleton structure. This structure has been used in [4] for the introduction of a novel stochastic scalar (univariate) Functional Model Based Method for the detection, localization, and magnitude (size) estimation of damages simulated by small masses added on the structure. In the present paper three scalar methods, namely a Power Spectral Density (PSD), a Frequency Response Function (FRF), and a model residual variance based method, as well as two vector methods, namely a model parameter based and a residual likelihood function based method, are employed, while the damages correspond to loosening of various bolts connecting the structural elements.

The main issues the study addresses include the following:

- (a) Comparison of the performance of *scalar* and *vector* statistical time series methods with regard to effective damage diagnosis; false alarms, missed damage and damage misclassification rates are investigated under multiple experiments.
- (b) Assessment of the methods in terms of their damage detection capability under various scenarios; *multiple* vibration measurement locations, “local” or “remote” to damage, are employed.
- (c) Assessment of the ability of the methods to accurately identify the actual damage type using “local” or “remote” sensors.
- (d) Discussion and assessment of the various methods features and facets.

**TABLE 1.** EXPERIMENTAL DETAILS

Structural State	Description	No of Experiments
Healthy	—	60
Damage A	loosening of bolts A1, A4, Z1, Z2	40
Damage B	loosening of bolts D1, D2, D3	40
Damage C	loosening of bolts K1	40
Damage D	loosening of bolts D2, D3	40
Damage E	loosening of bolts D3	40
Damage F	loosening of bolts K1, K2	40
Sampling frequency: $f_s = 512$ Hz, Signal bandwidth: $[4 - 200]$ Hz		
Signal length $N$ in samples (s):		
Non-parametric methods: $N = 46\ 080$ (90 s)		
Parametric methods: $N = 15\ 000$ (29 s)		

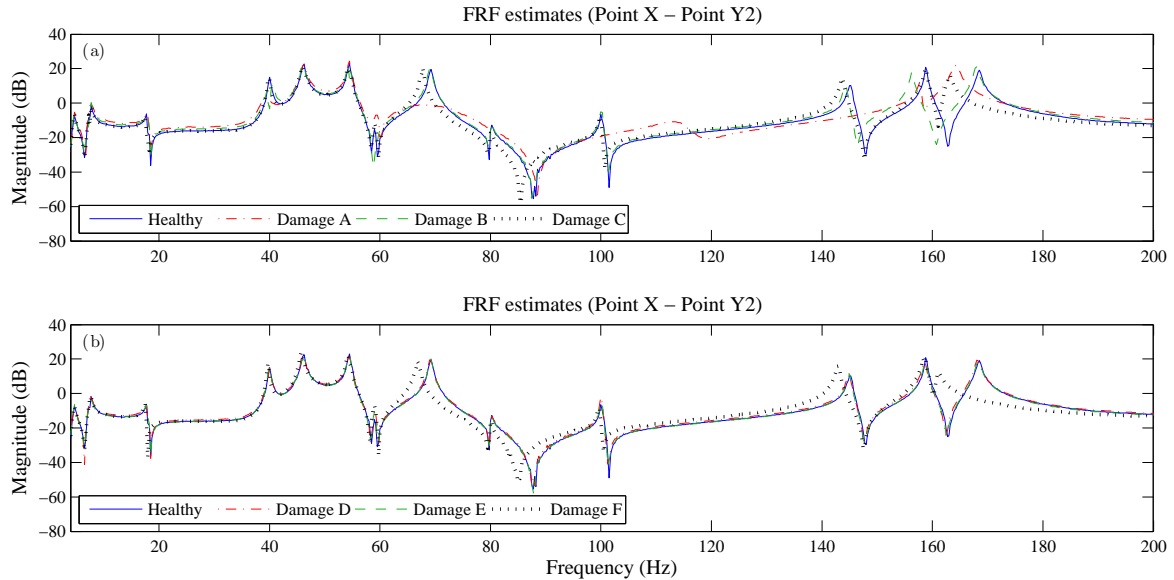
## THE STRUCTURE AND THE EXPERIMENTAL SET-UP

### The structure

The scale aircraft structure considered was designed by ON-ERA in conjunction with the GARTEUR SM-AG19 Group and manufactured at the University of Patras (Fig. 1). It represents a typical aircraft skeleton design and consists of six solid beams with rectangular cross sections representing the fuselage ( $1500 \times 150 \times 50$  mm), the wing ( $2000 \times 100 \times 10$  mm), the horizontal ( $300 \times 100 \times 10$  mm) and vertical stabilizers ( $400 \times 100 \times 10$  mm), and the right and left wing-tips ( $400 \times 100 \times 10$  mm). All parts are constructed from standard aluminum and are jointed together via steel plates and bolts. The total mass of the structure is approximately 50 kg.

### The damage types and the experiments

Damage detection and identification are based on vibration testing of the structure, which is suspended through a set of



**FIGURE 2.** FREQUENCY RESPONSE FUNCTION (FRF) MAGNITUDE ESTIMATES FOR THE HEALTHY AND DAMAGED STRUCTURAL STATES (POINT X – POINT Y2 TRANSFER FUNCTION).

**TABLE 2.** NON-PARAMETRIC ESTIMATION DETAILS

Data length	$N = 46\,080$ samples ( $\approx 90$ s)
Method	Welch
Segment length	$L = 2048$ samples
Non-overlapping segments	$K = 22$ segments
Window type	Hamming
Frequency resolution	$\Delta f = 0.25$ Hz

bungee cords and hooks from a long rigid beam sustained by two heavy-type stands (Fig. 1). The suspension is designed in a way to exhibit a pendulum rigid body mode below the frequency range of interest, as the boundary conditions are free-free.

The excitation is broadband random stationary Gaussian applied vertically at the right wing-tip (Point X, Fig. 1) through an electromechanical shaker (MB Dynamics Modal 50A, max load 225 N). The actual force exerted on the structure is measured via an impedance head (PCB M288D01), while the resulting vertical acceleration responses at Points Y1, Y2, Y3 and Y4 (Fig. 1) are measured via lightweight accelerometers (PCB 352A10 ICP). The force and acceleration signals are driven through a conditioning charge amplifier (PCB 481A02) into the data acquisition system based on SigLab 20-42 measurement modules.

The damage considered corresponds to the loosening of a variable number of bolts at different joints of the structure (Fig. 1). Six distinct types are considered and summarized in Tab. 1.

The assessment of the presented statistical time series methods with respect to the damage detection and identification sub-

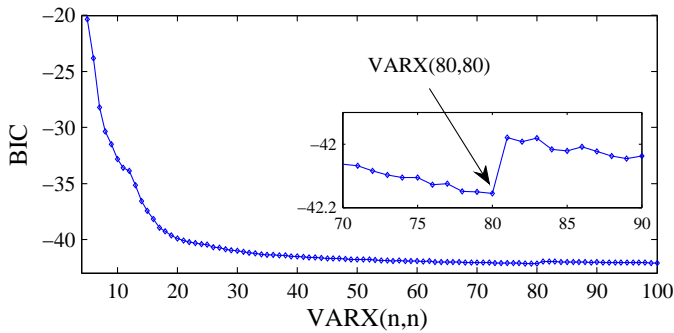
problems is based on 60 experiments for the healthy and 40 experiments for each considered damaged state of the structure (damage types A, B, ..., F – see Tab. 1). Moreover, *four* vibration measurement locations (Fig. 1, Points Y1 – Y4) are employed in order to determine the ability of the considered methods in treating damage diagnosis using single and multiple vibration response signals.

For damage detection a single healthy data set is used for establishing the baseline (reference) set, while 60 healthy and 240 damaged sets (six damage types with 40 experiments each) are used as inspection data sets. For the damage identification task, a single data set for each damaged structural state (damage types A, B, ..., F) is used for establishing the baseline (reference) set, while 240 sets are considered as inspection data sets (corresponding to unknown structural states). The time series models are estimated and the corresponding estimates of the characteristic quantity  $Q$  are extracted ( $\hat{Q}_A, \hat{Q}_B, \dots, \hat{Q}_F$  in the baseline phase;  $\hat{Q}_u$  in the inspection phase). Damage identification is presently based on successive binary hypothesis tests – as opposed to proper multiple hypothesis tests – and should be thus considered as preliminary [2].

## STRUCTURAL DYNAMICS OF THE HEALTHY STRUCTURE

### Non-parametric identification

Non-parametric identification of the structure is based on  $N = 46\,080$  ( $\approx 90$  s) sample-long excitation-response signals



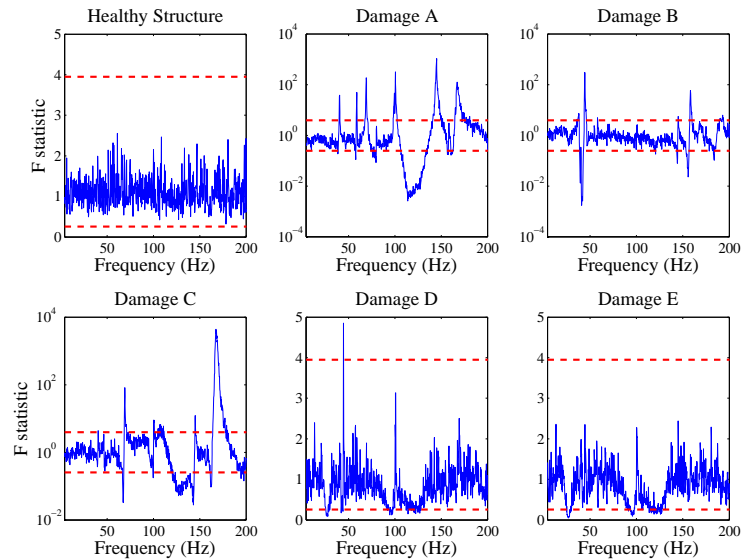
**FIGURE 3.** BAYESIAN INFORMATION CRITERION (BIC) FOR VARX( $n,n$ ) TYPE PARAMETRIC MODELS IN THE HEALTHY CASE.

obtained from *four* vibration measurement locations on the structure (see Fig. 1). An  $L = 2048$  sample-long Hamming data window with zero overlap is used (number of segments  $K = 22$ ) for PSD (MATLAB function *pwelch.m*) and FRF (MATLAB function *tfestimate.m*) Welch based estimation (see Tab. 2).

The obtained spectral estimates for the healthy and damaged states of the structure for the Point X – Point Y2 transfer function are depicted in Fig. 2. As it may be observed the FRF magnitude curves are quite similar in the 4 – 60 Hz range; notice that this range includes the first five modes of the structure. Significant differences between the healthy and damage type C, D and E magnitude curves are observed in the range of 60 – 150 Hz, where the next four modes are included. Finally, in the range of 150 – 200 Hz another two modes are present, and discrepancies are more evident for damage types A, B, C and F. Notice that the FRF magnitude curves for damage types D and E are very similar to those of the healthy structure.

### Parametric identification

Parametric identification of the structural dynamics is based on  $N = 15\,000$  ( $\approx 29$  s) sample-long excitation and single response signals, used to estimate Vector AutoRegressive with eXogenous excitation (VARX) models (MATLAB function *arx.m*). The modeling strategy consists of the successive fitting of VARX( $na,nb$ ) models (with  $na,nb$  designating the AR and X orders, respectively –  $na = nb = n$  is currently used) until a candidate model is selected. Model parameter estimation is achieved by minimizing a quadratic Prediction Error (PE) criterion (trace of residual covariance matrix) leading to a Least Squares (LS) estimator [12], [13, p. 206]. Model order selection, which is crucial for successful identification, may be based on a combination of tools, including the Bayesian Information Criterion (BIC) (Fig. 3), which is a statistical criterion that penalizes model complexity (order) as a counteraction to a decreasing model fit criterion [12], [13, pp. 505–507] and use of “stabilization diagrams” which depict the estimated modal parameters (usually



**FIGURE 4.** PSD BASED METHOD: INDICATIVE DAMAGE DETECTION RESULTS (OUTPUT 3) AT THE  $\alpha = 10^{-5}$  RISK LEVEL. THE ACTUAL STRUCTURAL STATE IS SHOWN ABOVE EACH PLOT.

frequencies) as a function of increasing model order [12, 13]. BIC minimization is achieved for model order  $n = 80$  (Fig. 3), thus a 4–variate VARX(80, 80) model is selected as adequate for the model parameter, residual variance, and likelihood function based methods. The identified VARX(80, 80) representation has 1604 parameters, yielding a Sample Per Parameter (SPP) number equal to 37.4.

### SCALAR TIME SERIES METHODS FOR SHM

Time series methods for SHM employ *scalar* (univariate case) or *vector* (multivariate case) random vibration excitation–response signals. The multivariate case requires the establishment of vector statistics and the use of corresponding models [14]. Despite their phenomenal resemblance to their univariate counterparts, multivariate models generally have a much richer structure, while they typically require multivariate statistical decision making procedures [2, 14].

In this section, two non-parametric, namely a PSD and an FRF based method, and a parametric (residual variance based method) *scalar* time series method for SHM are briefly reviewed, and corresponding results are presented and discussed. The main characteristics of the methods are summarized in Tab. 3.

### A Power Spectral Density (PSD) based method

Damage detection and identification is in this case tackled via characteristic changes in the Power Spectral Density (PSD) of the measured vibration response signals (non-parametric method). The excitation is not assumed available (*response-only*

**TABLE 3. CHARACTERISTICS OF STATISTICAL TIME SERIES METHODS FOR SHM**

Method	Principle	Test Statistic	Type
PSD based	$S_u(\omega) \stackrel{?}{=} S_o(\omega)$	$F = \widehat{S}_o(\omega)/\widehat{S}_u(\omega) \sim F(2K, 2K)$	scalar
FRF based	$\delta H(j\omega)  =  H_o(j\omega)  -  H_u(j\omega)  \stackrel{?}{=} 0$	$Z = \delta \widehat{H}(j\omega) /\sqrt{2}\widehat{\sigma}_H \sim N(0, 2\sigma_H^2(\omega))$	scalar
Residual variance	$\sigma_{oo}^2 \stackrel{?}{\geq} \sigma_{ou}^2$	$F = \widehat{\sigma}_{ou}^2/\widehat{\sigma}_{oo}^2 \sim F(N, N-d)$	scalar
Model parameter	$\delta\theta = \theta_o - \theta_u \stackrel{?}{=} 0$	$\chi_\theta^2 = \delta\widehat{\theta}^T (2\widehat{P}_\theta)^{-1} \delta\widehat{\theta} \sim \chi^2(d)$	vector
Residual likelihood	$\theta_o \stackrel{?}{=} \theta_u$	$\sum_{t=1}^N (e_u^T[t, \theta_o] \cdot \Sigma_o \cdot e_u[t, \theta_o]) \leq l$	vector

$S(\omega)$ : Power Spectral Density (PSD) function;  $|H(j\omega)|$ : Frequency Response Function (FRF) magnitude

$\sigma_H$ : standard deviation of  $|\widehat{H}_o(j\omega)|$ ;  $\theta$ : model parameter vector;  $d$ : parameter vector dimensionality;  $P_\theta$ : covariance of  $\theta_o$

$\sigma_{oo}^2$ : variance of residual signal obtained by driving the healthy structure signals through the healthy model

$\sigma_{ou}^2$ : variance of residual signal obtained by driving the current structure signals through the healthy model

$e$ :  $k$ -variate residual sequence;  $\Sigma$ : residual covariance matrix;  $l$ : user defined threshold;  $N$ : signal length in samples

In all cases estimators/estimates are designated by a hat.

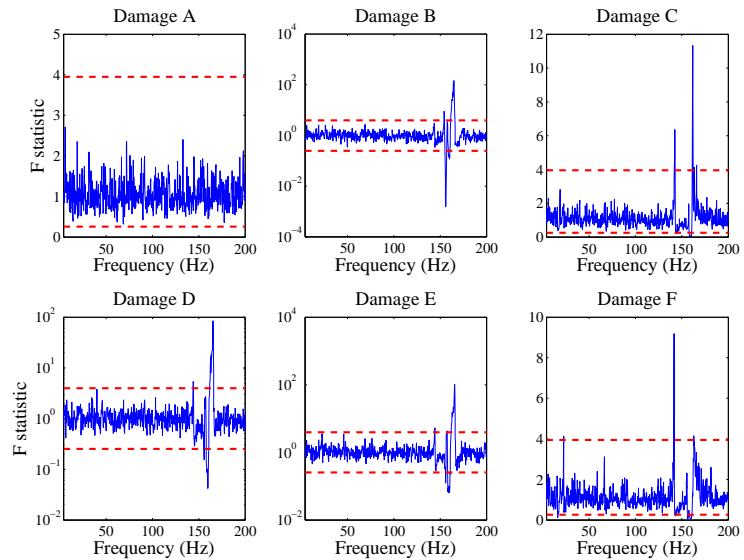
The subscripts “o” and “u” designate healthy and current (unknown) structural state, respectively.

case). The method’s characteristic quantity thus is  $Q = S(\omega)$  ( $\omega$  designates frequency) (see Tab. 3). Damage detection is based on confirmation of statistically significant deviations (from the nominal/healthy) in the current structure’s PSD function at some frequency [1, 2]. Damage identification may be achieved by performing hypothesis testing similar to the above separately for damages of each potential type. It should be noted that response signal scaling is important in order to properly account for potentially different excitation levels.

**Results.** Typical non-parametric damage detection results obtained from vibration measurement location at Point Y3 (output 3) are presented in Fig. 4. Evidently, correct detection at the  $\alpha = 10^{-5}$  risk level is obtained in each case, as the test statistic is shown not to exceed the critical points (dashed horizontal lines) in the healthy case, while it exceeds it in each damage case. Observe that damage types A, B and C (see Fig. 1 and Tab. 1) appear more severe (note the logarithmic scale on the vertical axis of Fig. 4), while damage types D and E are harder to detect.

Representative damage identification results at the  $\alpha = 10^{-5}$  risk level for vibration measurement location at Point Y1 (output 1) are presented in Fig. 5, with the actual damage being of type A. The test statistic does not exceed the critical points in the first case, while this is exceeded in the remaining cases. This correctly identifies damage type A as current.

Summary damage detection and identification results for the considered vibration measurement locations (Fig. 1) are presented in Tab. 4. The PSD based method achieves accurate damage detection as no false alarms are exhibited, while the number of missed damage cases is zero for all considered damaged structural states. The method is also capable of identifying the actual damage type, as zero damage misclassification errors were reported for damage types A, C, D and F, while it exhibits some misclassification errors for damage type E. The misclassification

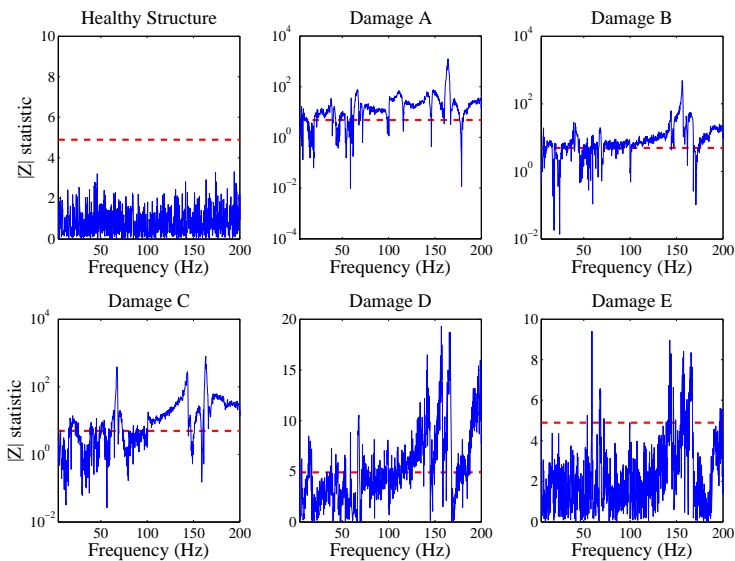


**FIGURE 5. PSD BASED METHOD: INDICATIVE DAMAGE IDENTIFICATION RESULTS (OUTPUT 1) AT THE  $\alpha = 10^{-5}$  RISK LEVEL, WITH THE ACTUAL DAMAGE BEING OF TYPE A. EACH CONSIDERED TEST CASE IS SHOWN ABOVE EACH PLOT.**

problem is more intense for damage type B from the Y3 and Y4 vibration measurement locations (Tab. 4).

### A Frequency Response Function (FRF) based method

This is similar to the previous method, except that it requires the availability of both the excitation and response signals (*excitation-response case*) and uses the FRF magnitude as its characteristic quantity (non-parametric method), thus  $Q = |H(j\omega)|$  with  $j = \sqrt{-1}$  (see Tab. 3). The main idea is the comparison of the FRF magnitude  $|H_u(j\omega)|$  of the current state of the



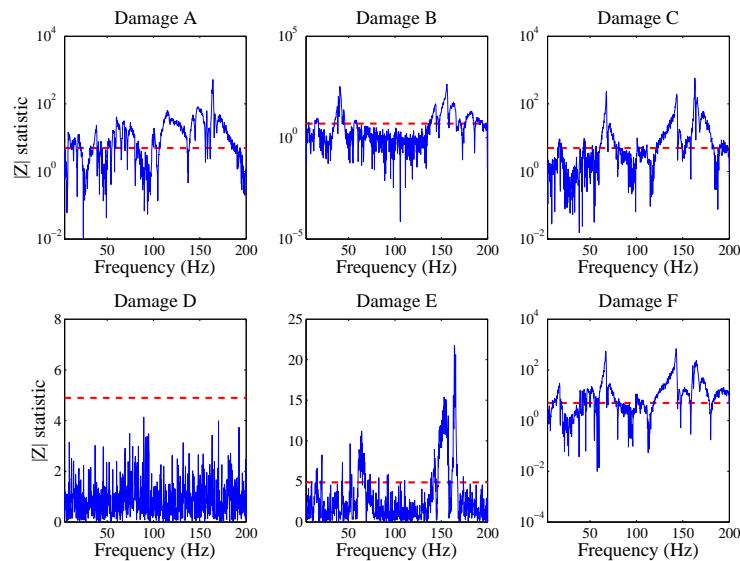
**FIGURE 6.** FRF MAGNITUDE BASED METHOD: INDICATIVE DAMAGE DETECTION RESULTS (OUTPUT 2) AT THE  $\alpha = 10^{-6}$  RISK LEVEL. THE ACTUAL STRUCTURAL STATE IS SHOWN ABOVE EACH PLOT.

structure to that of the healthy structure  $|H_o(j\omega)|$ . Damage detection is based on confirmation of statistically significant deviations (from the nominal/healthy) in the current structure's FRF at one or more frequencies through a hypothesis testing problem (for each  $\omega$ ) [1, 2]. Damage identification may be achieved by performing hypothesis testing similar to the above separately for damages of each potential type.

**Results.** Figure 6 presents typical non-parametric damage detection results via the FRF based method obtained at vibration measurement location Y2 (output 2). Evidently, correct detection at the  $\alpha = 10^{-6}$  risk level is achieved in each case, as the test statistic is shown not to exceed the critical points (dashed horizontal lines) in the healthy case, while it exceeds the critical point in the damaged cases. Again, damage types A, B and C are the more severe, while damage types D and E are harder to detect.

Indicative damage identification results at the  $\alpha = 10^{-6}$  risk level for output 4 via the FRF based method are presented in Fig. 7, with the actual damage being of type D. The test statistic does not exceed the critical point in this (Damage D) case, while it exceeds it in all other cases. This correctly identifies damage type D as current.

The summarized damage detection and identification results for the considered vibration measurement locations (Fig. 1) are presented in Tab. 4. The FRF magnitude based method achieves effective damage detection as no false alarms or missed damages are reported (Tab. 4). The method on the other hand, exhibits decreased accuracy in damage identification as significant numbers



**FIGURE 7.** FRF MAGNITUDE BASED METHOD: INDICATIVE DAMAGE IDENTIFICATION RESULTS (OUTPUT 3) AT THE  $\alpha = 10^{-6}$  RISK LEVEL, WITH THE ACTUAL DAMAGE BEING OF TYPE D. EACH CONSIDERED TEST CASE IS SHOWN ABOVE EACH PLOT.

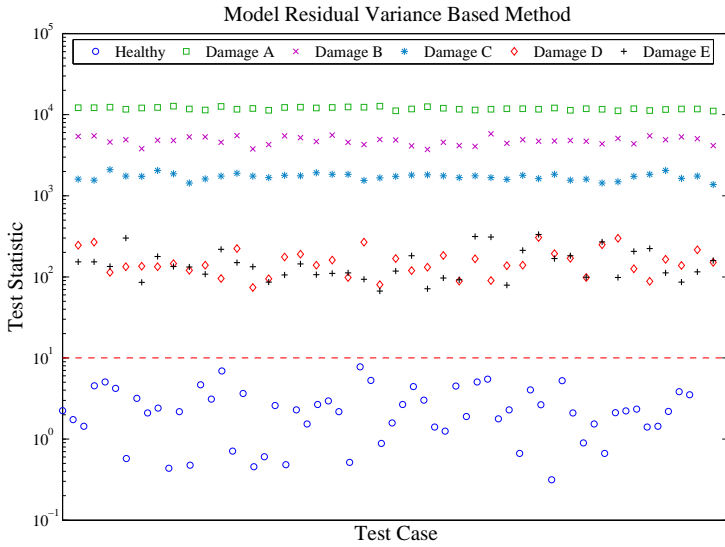
of damage misclassification errors are reported for damage types B and D (Tab. 4).

### Residual variance based method

In this method (*excitation-response case*) the characteristic quantity is the residual variance. The main idea is based on the fact that the model (parametric method) matching the current state of the structure should generate a residual sequence characterized by minimal variance [1, 2]. Damage detection is based on the fact that the residual series obtained by driving the current signal(s) through the model corresponding to the nominal (healthy) structure have variance that is minimal if and only if the current structure is healthy [1, 2]. This method uses classical tests on the residuals and offers simplicity and no need for model estimation in the inspection phase. The method's main characteristics are shown in Tab. 3.

**Results.** The residual variance based method is based on the identified 4-variate VARX(80, 80) models obtained from the baseline phase, as well as on corresponding models from the current (unknown) data records (inspection phase). Damage detection and identification is achieved via statistical comparison of the two residual variances (observe that each one of the scalar responses is considered separately).

Typical damage detection and identification results obtained via the residual variance based method for vibration measurement location Y2 are shown in Fig. 8 and Fig. 9. Evidently, correct detection (Fig. 8) is obtained in each considered case, as the test statistic is shown not to exceed the critical point in the



**FIGURE 8.** RESIDUAL VARIANCE BASED METHOD: INDICATIVE DAMAGE DETECTION RESULTS (OUTPUT 2; HEALTHY – 60 EXPERIMENTS; DAMAGED – 200 EXPERIMENTS). A DAMAGE IS DETECTED IF THE TEST STATISTIC EXCEEDS THE CRITICAL POINT (DASHED HORIZONTAL LINE).

healthy case, while it exceeds it in the damaged test cases. Moreover, Fig. 9 demonstrates the ability of the method to correctly identify the actual damage type.

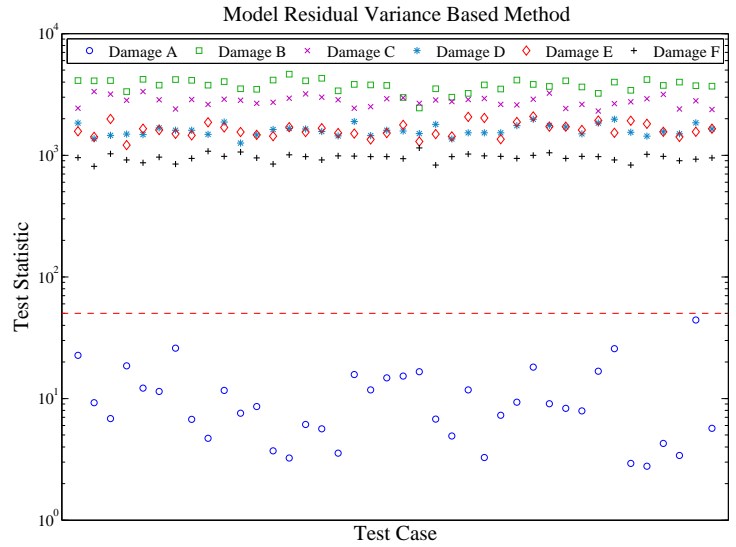
Summary damage detection and identification results for the considered vibration measurement locations (Fig. 1) are presented in Tab. 4. The method achieves effective damage detection and identification as no false alarms, missed damages, or damage misclassification cases are observed.

### VECTOR TIME SERIES METHODS FOR SHM

Two *vector* (multivariate) parametric time series methods for SHM, namely a model parameter based method and a residual likelihood function based method, are presently reviewed, while their experimental results are presented and assessed. The main characteristics of the methods are summarized in Tab. 3.

#### A model parameter based method

This method bases damage detection and identification on a characteristic quantity  $Q = \theta$  which is function of the parameter vector  $\theta$  of a parametric time series model (parametric method) [1, 2]. In this method the model has to be re-estimated in the inspection phase based on signals from the current (unknown) state of the structure. Damage detection is based on testing for statistically significant changes in the parameter vector  $\theta$  between the nominal and current structures through a hypothesis



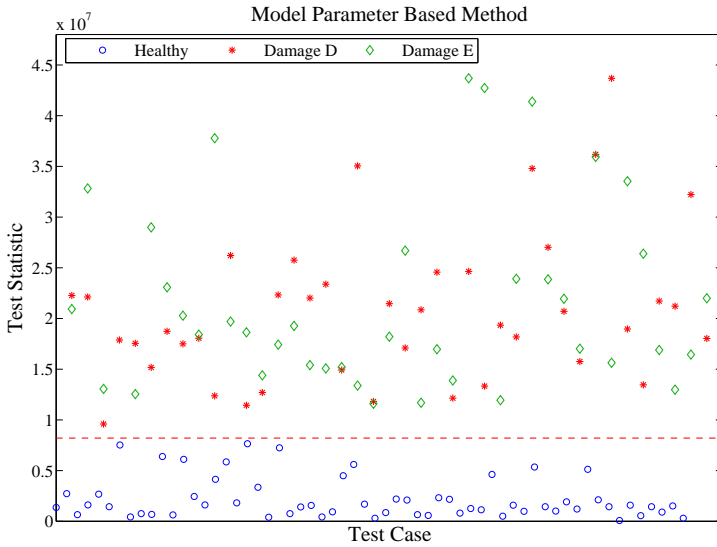
**FIGURE 9.** RESIDUAL VARIANCE BASED METHOD: INDICATIVE DAMAGE IDENTIFICATION RESULTS (OUTPUT 2; 240 EXPERIMENTS), WITH THE ACTUAL DAMAGE BEING OF TYPE A. A DAMAGE IS IDENTIFIED AS TYPE A IF THE TEST STATISTIC IS BELOW THE CRITICAL POINT (DASHED HORIZONTAL LINE).

testing problem. Damage identification may be based on multiple hypothesis testing comparing the current parameter vector to those corresponding to different damage types. In the present case a procedure that uses a series of binary hypothesis tests is employed. The method's main characteristics are presented in Tab. 3.

**Results.** The model parameter based method (*excitation-response case*) employs the identified in the baseline phase 4-variate VARX(80,80) models, as well as an identified VARX(80,80) model for each current data record (inspection phase).

Figure 10 presents typical parametric damage detection results. The healthy test statistics are shown in circles (60 experiments), while the least severe damage types D and E are presented with asterisks and diamonds, respectively (one for each one of the 40 experiments). Evidently, correct detection is obtained in each case, as the test statistic is shown not to exceed the critical point in the healthy cases, while it exceeds it in the damaged cases; note the logarithmic scale on the vertical axis which indicates significant difference between the healthy and damaged test statistics for the considered test cases.

As Tab. 5 indicates, the model parameter based method achieves accurate damage detection and identification, as no false alarm, missed damage, or damage misclassification cases are reported.



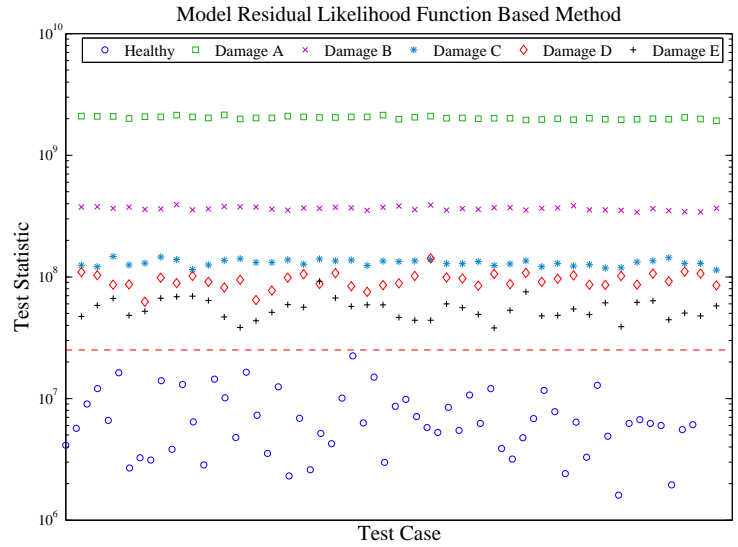
**FIGURE 10.** MODEL PARAMETER BASED METHOD: INDICATIVE DAMAGE DETECTION RESULTS FOR THREE STRUCTURAL STATES (HEALTHY – 60 EXPERIMENTS; DAMAGED – 80 EXPERIMENTS). A DAMAGE IS DETECTED IF THE TEST STATISTIC EXCEEDS THE CRITICAL POINT (DASHED HORIZONTAL LINE).

### Residual likelihood function based method

In this parametric method, damage detection is based on the likelihood function evaluated for the current signal(s) under each one of the considered structural states [1, 2], [15, pp. 119–120]. The hypothesis corresponding to the largest likelihood is selected as true for the current structural state. Damage identification is achieved by computing the likelihood function of the current signal(s) for the baseline models corresponding to damaged structural states and accepting the hypothesis that corresponds to the maximum value of the likelihood – by including the healthy baseline model damage detection is also treated. This method offers simplicity as there is no need for model estimation in the inspection phase. The method’s main characteristics are shown in Tab. 3.

**Results.** The residual likelihood function based method (*excitation–response case*) is based on the identified 4–variate VARX(80,80) models from the baseline phase. Figure 11 presents typical damage detection results obtained by the likelihood function based method. Evidently, correct detection is obtained in each case, as the test statistic is shown not to exceed the critical point in the healthy cases, while it exceeds it in the damaged cases. Indicative damage identification results, with the actual damage being of type C, are depicted in Fig. 12.

The method achieves accurate damage detection and identification, as no false alarm, missed damage, or damage misclassification cases are reported. Summary damage detection and identification results are presented in Tab. 5.



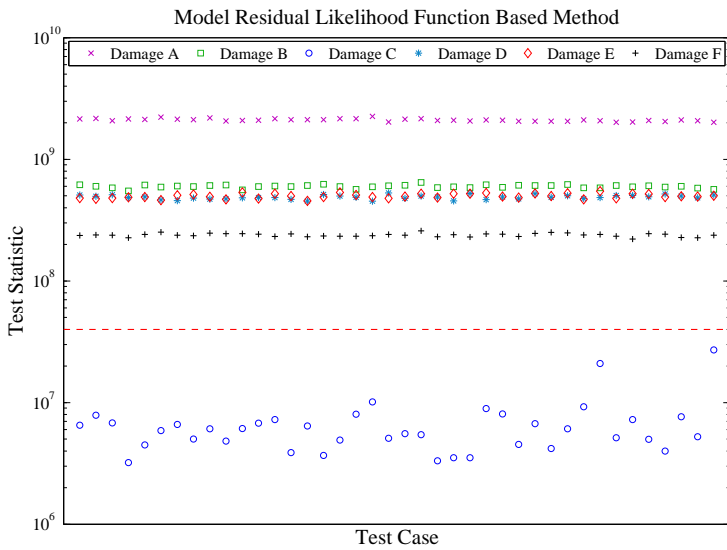
**FIGURE 11.** RESIDUAL LIKELIHOOD FUNCTION BASED METHOD: INDICATIVE DAMAGE DETECTION RESULTS (HEALTHY – 60 EXPERIMENTS; DAMAGED – 200 EXPERIMENTS). A DAMAGE IS DETECTED IF THE TEST STATISTIC EXCEEDS THE CRITICAL POINT (DASHED HORIZONTAL LINE).

### DISCUSSION

*Scalar* time series methods for SHM are shown to achieve effective damage detection and identification, although non–parametric scalar methods encounter some difficulties. The PSD based method achieves excellent damage detection, although it exhibits some misclassification errors for damage type E. The misclassification problem is more intense for damage type B and the Y3 and Y4 vibration measurement locations. The FRF based method achieves accurate damage detection with no false alarms or missed damages, except for vibration measurement location Y4 for which it exhibits an increased number of false alarms. Moreover, it faces problems in correctly identifying damage types B and D, as the number of damage misclassification cases is higher for these specific damage types. Both of these damage types involve loosening of bolts on the left wing–tip of the aircraft (Fig. 1). On the other hand, the parametric residual variance based method achieves excellent performance in accurately detecting and identifying damage for all considered vibration measurement locations (Tab. 4).

*Vector* time series methods for SHM achieve very accurate damage detection and identification, as with properly adjusted risk level  $\alpha$  (type I error) no false alarm, missed damage, or damage misclassification cases are reported. Moreover, the methods demonstrate global damage detection capability. Nevertheless, parametric vector models require accurate parameter estimation and appropriate model structure (order) selection in order to accurately represent the structural dynamics and effectively tackle





**FIGURE 12.** RESIDUAL LIKELIHOOD FUNCTION BASED METHOD: INDICATIVE DAMAGE IDENTIFICATION RESULTS (240 EXPERIMENTS), WITH THE ACTUAL DAMAGE BEING OF TYPE C. A DAMAGE IS IDENTIFIED AS TYPE C IF THE TEST STATISTIC IS BELOW THE CRITICAL POINT (DASHED HORIZONTAL LINE).

the damage detection and identification subproblems. Therefore, methods falling into this category require adequate user expertise and are somewhat more elaborate than their scalar or non-parametric counterparts.

Furthermore, the number and location of vibration measurement sensors is an important issue. Several vibration based damage diagnosis techniques that appear to work well in certain test cases, could actually perform poorly when subjected to the measurement constraints imposed by actual testing [3]. Techniques that are to be seriously considered for implementation in the field should demonstrate that they can perform well under limitations of a small number of measurement locations and under the constraint that these locations should be selected a-priori, without knowledge of the actual damage location. In the present study, statistical time series methods were demonstrated to be capable of achieving effective damage diagnosis based on very limited (*vector case*), or even on a single-pair (*scalar case*), of excitation-response measurements. Nevertheless, their performance on large scale structures should be further investigated.

Moreover, in order for certain parametric methods to work effectively, a very small value of the type I risk  $\alpha$  is often needed. This is due to the fact that the current stochastic time series models (ARMA, ARX, State Space and so on) used for modeling the structural dynamics are incapable of fully capturing the experimental, operational and environmental uncertainties that the structure is subjected to. For this reason, a very small  $\alpha$  is often selected in order to compensate for the lack of effective uncer-

tainty modeling. More accurate modeling of uncertainties is an important subject of current research – in this context see [16].

## CONCLUDING REMARKS

- Statistical time series methods for SHM achieve effective damage detection and identification based on (i) random excitation and/or vibration response (*scalar* or *vector*) signals, (ii) statistical model building, and (iii) statistical decision making under uncertainty.
- Both scalar and vector statistical time series methods for SHM were shown to effectively tackle damage detection and identification, with the vector methods achieving excellent performance with zero false alarm, missed damage and damage misclassification rates.
- Both scalar and vector methods have global damage detection capability, as they are able to detect “local” and “remote” damage with respect to the sensor location being used.
- All methods were able to correctly identify the actual damage type, with the exception of the FRF based method which exhibited an increased number of damage misclassification errors for the two damage types that affect the left wing-tip of the aircraft scale skeleton structure.
- Parametric time series methods are more elaborate and require higher user expertise compared to their generally simpler non-parametric counterparts. Yet, they offer increased sensitivity and accuracy. Moreover, vector methods based on multivariate models are more elaborate but offer the potential of further enhanced performance.
- The availability of data records corresponding to various potential damage scenarios is necessary in order to treat damage identification. This may not be possible with the actual structure itself, but laboratory scale models or analytical (Finite Element) models may be used for this purpose.

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**TABLE 4. SCALAR METHODS DAMAGE DETECTION AND IDENTIFICATION SUMMARY RESULTS**

Method	Damage Detection							Damage Identification					
	False alarms	Missed damage						Damage misclassification					
		dam. A	dam. B	dam. C	dam. D	dam. E	dam. F	dam. A	dam. B	dam. C	dam. D	dam. E	dam. F
<b>PSD based</b>													
response Y1	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y2	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y3	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	<b>21/40</b>	0/40	1/40	0/40
response Y4	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	<b>21/40</b>	0/40	2/40	0/40
<b>FRF based</b>													
response Y1	1/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	<b>10/40</b>	<b>6/40</b>	<b>5/40</b>	2/40	0/40
response Y2	0/60	0/40	0/40	0/40	0/40	1/40	0/40	0/40	<b>4/40</b>	<b>10/40</b>	<b>22/40</b>	<b>9/40</b>	3/40
response Y3	0/60	0/40	0/40	0/40	1/40	0/40	0/40	0/40	<b>7/40</b>	2/40	<b>9/40</b>	<b>5/40</b>	1/40
response Y4	<b>35/60</b>	0/40	0/40	0/40	0/40	0/40	0/40	0/40	<b>8/40</b>	0/40	<b>8/40</b>	2/40	0/40
<b>Res. variance<sup>†</sup></b>													
response Y1	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y2	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y3	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y4	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40

<sup>†</sup> adjusted  $\alpha$

**TABLE 5. VECTOR METHODS DAMAGE DETECTION AND IDENTIFICATION SUMMARY RESULTS**

Method	Damage Detection							Damage Identification					
	False alarms	Missed damage						Damage misclassification					
		dam. A	dam. B	dam. C	dam. D	dam. E	dam. F	dam. A	dam. B	dam. C	dam. D	dam. E	dam. F
Mod. parameter <sup>†</sup>	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
Res. likelihood <sup>†</sup>	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40

<sup>†</sup> adjusted  $\alpha$

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