Experimental Assessment of Vibration–Based Time Series Methods for Structural Health Monitoring

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ABSTRACT

This work aims at the experimental assessment of vibration–based time series methods for Structural Health Monitoring (SHM). The methods are classified as non–parametric and parametric, while their features and operation are discussed. Furthermore, their performance characteristics and ease of use are assessed and compared. The application and comparative assessment of the methods through experiments on a laboratory aluminum truss structure is presented. The results of the study confirm the high potential and effectiveness of the statistical time series methods for SHM.

INTRODUCTION

Vibration–based statistical time series methods for damage detection and identification utilize random excitation and/or vibration response signals, along with statistical model building and decision making tools, for inferring the health state of a structure [1–4]. They offer a number of important advantages, including no requirement for physics–based or finite element type models, no requirement for complete modal models, effective treatment of uncertainties, and statistical decision making with specified performance characteristics. In spite of the above, the literature on vibration–based time series methods for SHM remains relatively sparse, and, in particular, no application studies that assess and compare the various methods are available.

The *goal* of this study is to contribute to filling this gap by presenting the application and comparative assessment of a number of statistical time methods to a laboratory aluminum truss structure. The damages considered correspond to the loosening of bolts connecting certain of the truss elements. Random force excitation is provided via an electromechanical shaker, while the vibration responses are measured at various positions via lightweight strain gauges. Two non–parametric methods (Power

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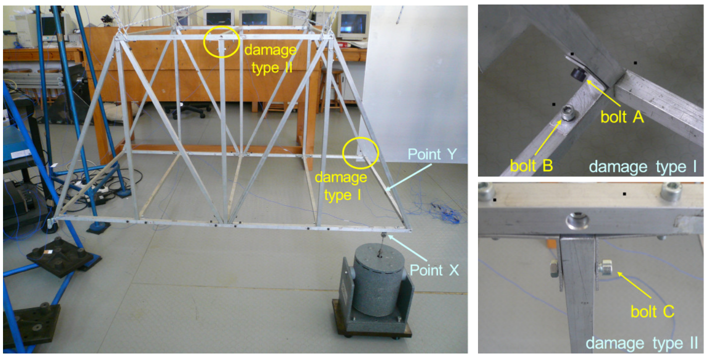

Figure 1. The aluminum truss structure and the experimental set-up: The force excitation (point X), the vibration measurement position (Point Y), and the damage types I and II.

Spectral Density and Frequency Response Function based) and four parametric methods (model parameter based, residual variance, likelihood function, residual uncorrelatedness based) are employed.

THE EXPERIMENTAL SET-UP

The structure. The truss structure employed was designed and manufactured by the SMSA Laboratory of the University of Patras. It consists of thirty beam elements with rectangular cross sections jointed together via steel elbow plates and bolts (Figure 1). All parts are constructed from standard aluminum with the truss dimensions being $140 \times 70 \times 80$ mm.

The damage and the experiments. The damage considered corresponds to loosening of a variable number of bolts at different joints of the truss. Two distinct types are considered: The first damage type, referred to as *damage type I*, corresponds to loosening of two bolts joining together an horizontal and a vertical beam element (bolts A and B, Figure 1). The second damage type, referred to as *damage type II*, corresponds to the loosening of one bolt (bolt C, Figure 1).

Damage detection and identification are based on vibration testing of the structure, which is suspended through a set of cords. The excitation is a random Gaussian force applied vertically at Point X (Figure 1) via an electromechanical shaker (MB Dynamics Modal 50A). The actual force exerted on the structure is measured via an impedance head (PCB 288D01), while the resulting responses are measured at different points on the structure via strain gauges (PCB ICP 740B02). In this study the results based on a single response measurement (Point Y, Figure 1) are presented. The force and strain signals are driven through a signal conditioning device (PCB 481A02) into the data acquisition system (SigLab 20-42).

A number of experiments are carried out, initially for the healthy and subsequently for the damaged states of the structure. Experimental details are presented in Table I. In Figure 2 the non–parametric (Welch based) and parametric (AutoRegressive with eXogenous excitation – ARX – model based) Frequency Response Function (FRF) estimates for the healthy and damaged states of the structure are depicted. As it may be observed, damage type I is more evident than damage type II.

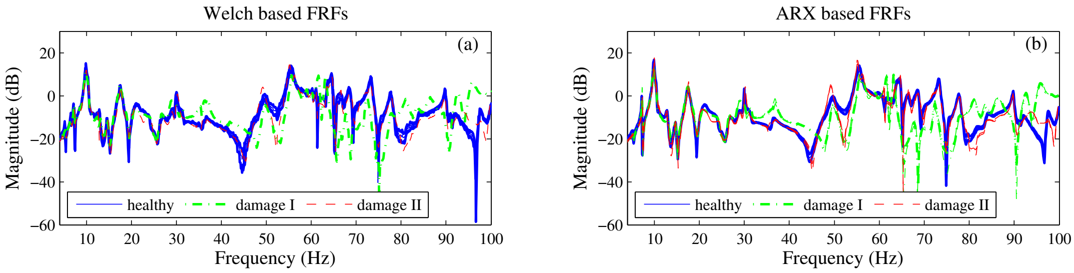

Figure 2. Frequency Response Function (FRF) magnitude estimates for the healthy and damaged states of the structure: (a) Non-parametric (Welch based) estimates and (b) parametric (ARX based) estimates. For the healthy FRFs the ± 2 standard deviation confidence intervals are also depicted (solid thick lines).

STATISTICAL TIME SERIES METHODS FOR DAMAGE DETECTION AND IDENTIFICATION

An extensive overview of time series methods for SHM in vibrating structures can be found in [1]. In this study only the basic principles of the methods are reviewed. The methods are based on the detection of changes in a characteristic quantity Q constructed from each data set under the healthy/nominal and damaged structural states. Depending on the way this characteristic quantity is constructed, time series methods may be classified as non–parametric or parametric. A rough comparison of the methods is presented in Table II, while their main characteristics are presented in Table III.

Non–parametric methods. Non–parametric methods (see [1] and the references therein) are those in which the characteristic quantity Q is constructed based on non–parametric time series representations (models). Two non–parametric methods are briefly reviewed in the following: a Power Spectral Density (PSD) based method

TABLE I. EXPERIMENTAL DETAILS

Structural State	Description	No of Experiments
Healthy	—	40
Damage type I	loosening of bolts A and B	32
Damage type II	loosening of bolt C	32
Sampling frequency: $f_s = 256$ Hz, Signal bandwidth: $[0.5 - 100]$ Hz		
Signal length N in samples (s):		
Non-parametric methods: $N = 30720$ (120 s), Parametric methods: $N = 10000$ (39 s)		

TABLE II. COMPARISON OF NON–PARAMETRIC AND PARAMETRIC METHODS

Methods	*Advantages*	*Disadvantages*
Non-parametric	Simplicity	Potentially reduced accuracy
	Computational efficiency	
	Minimal user expertise required	
Parametric	Improved parsimony	Increased complexity
	Potentially increased accuracy	Computationally involved
		Increased user expertise required

and a Frequency Response Function (FRF) based method. For both methods the excitation and/or vibration response signals are segmented into K non–overlapping sections, each of length L, and Welch type spectral estimation is employed.

THE POWER SPECTRAL DENSITY (PSD) BASED METHOD

This method tackles damage detection and identification via characteristic changes in the PSD of the measured vibration signal when the excitation is not available (response–only case). The method's characteristic quantity thus is $Q = S(\omega)$ (ω designates frequency). Damage detection is based on confirmation of statistically significant deviations (from the nominal/healthy) in the current structure's PSD function at some frequency [1]. Damage identification may be achieved by performing hypotheses testing similar to the above separately for damages of each potential type.

THE FREQUENCY RESPONSE FUNCTION (FRF) BASED METHOD

This is similar to the previous method, except that it requires the availability of both the excitation and response signals and uses the FRF magnitude as its characteristic quantity, thus $Q = |H(j\omega)|$ with $j = \sqrt{-1}$. Damage detection is based on confirmation of statistically significant deviations (from the nominal/healthy) in the current structure's FRF at one or more frequencies through a hypothesis testing problem (for each ω) [1]. Damage identification may be achieved by performing hypotheses testing similar to the above separately for damages of each potential type.

Parametric methods. Parametric methods are those in which the characteristic quantity Q is constructed based on parametric time series representations (models) [5] (for details see [1] and the references therein). They are applicable to both the response–only and excitation–response cases, as each situation may be dealt with through the use of proper representations. The parametric methods reviewed in this study are the

TABLE III. OVERVIEW OF TIME SERIES METHODS FOR SHM

Method	Principle	Test Statistic
PSD based	$S_u(\omega) \stackrel{?}{=} S_o(\omega)$	$F = \widehat{S}_o(\omega)/\widehat{S}_u(\omega)$
FRF based	$\delta\|H(j\omega)\| = \|H_o(j\omega)\| - \|H_u(j\omega)\| \stackrel{?}{=} 0$	$Z = \delta\|\widehat{H}(j\omega)\|/\sqrt{2}\widehat{\sigma}_H$
Model parameter	$\delta\boldsymbol{\theta} = \boldsymbol{\theta}_o - \boldsymbol{\theta}_u \stackrel{?}{=} \mathbf{0}$	$\chi^2_\theta = \delta\widehat{\boldsymbol{\theta}}^T(2\widehat{\boldsymbol{P}}_\theta)^{-1}\delta\widehat{\boldsymbol{\theta}}$
Residual variance	$\sigma^2_{oo} \stackrel{?}{\geq} \sigma^2_{ou}$	$F = \widehat{\sigma}^2_{ou}/\widehat{\sigma}^2_{oo}$
Residual likelihood	$\boldsymbol{\theta}_o \stackrel{?}{=} \boldsymbol{\theta}_u$	$\chi^2_N = N\widehat{\sigma}^2_{ou}/\sigma^2_{oo}$
Residual uncorr.	$\rho[\tau] \stackrel{?}{=} 0$	$\chi^2_\rho = N(N+2)\sum_{\tau=1}^{r}(N-\tau)^{-1}\widehat{\rho}^2[\tau]$

$^\dagger S(\omega)$: Power Spectral Density (PSD) function; $|H(j\omega)|$: Frequency Response Function (FRF) magnitude
σ_H: standard deviation of $|\widehat{H}_o(j\omega)|$; $\boldsymbol{\theta}$: model parameter vector; $\boldsymbol{P}_\theta$: covariance of $\boldsymbol{\theta}_o$
σ^2_{oo}: variance of residual signal obtained by driving the healthy structure signals through the healthy model
σ^2_{ou}: variance of residual signal obtained by driving the current structure signals through the healthy model
$\rho[\tau]$: residual normalized autocovariance; N: signal length in samples
Estimators/estimates are designated by a hat.
The subscripts "o" and "u" designate healthy and current (unknown) structural state, respectively.

model parameter based method and certain residual based methods.

THE MODEL PARAMETER BASED METHOD

This method attempts damage detection and identification using a characteristic quantity Q that is function of the parameter vector $\boldsymbol{\theta}$ of a parametric time series model. In this method the model has to be re–estimated during the inspection phase based on signals from the current (unknown) state of the structure. Damage detection is based on testing for statistically significant changes in the parameter vector $\boldsymbol{\theta}$ between the nominal and current structures through a hypothesis testing problem [1]. Damage identification may be based on multiple hypotheses testing problems comparing the current parameter vector to those corresponding to different damage types.

RESIDUAL BASED METHODS

These methods [1] attempt damage detection and identification using characteristic quantities that are functions of residual sequences obtained by driving the current structural excitation and/or response signals through suitable pre–determined models corresponding to a particular state of the structure (healthy and damaged structure under specific damage types).

A first method is based on the fact that the model matching the current state of the structure should generate a residual sequence characterized by minimal variance. A second method is based on the likelihood function evaluated for the current signal(s) under each one of the considered structural states. The hypothesis corresponding to the largest likelihood is selected as true for the current structure. A third method is based on examination of the residual series uncorrelatedness. The model matching the current state of the structure should generate a white (uncorrelated) residual sequence. These methods use classical tests on the residuals and offer simplicity and no need for model estimation in the inspection phase.

EXPERIMENTAL RESULTS

A comparative assessment of the statistical time series methods is now presented. The non–parametric methods' details are presented in Table IV, while those of para-

TABLE IV. NON–PARAMETRIC METHODS DETAILS

Method	Segment length (L)	Non-overlapping segments (K)	Window type
Welch	2048 samples	15	Hamming

TABLE V. PARAMETRIC METHODS DETAILS

Method	Estimated Model	Dimension of $\boldsymbol{\theta}$	Max. Lag r
Model parameter	ARX$(103, 103)$	207 parameters	–
Residual variance	ARX$(103, 103)$	207 parameters	–
Residual likelihood function	ARX$(103, 103)$	207 parameters	–
Residual uncorrelatedness	ARX$(138, 138)$	277 parameters	25 samples
ARX(na, nb) stands for AutoRegressive model with eXogenous excitation of orders (na, nb)			

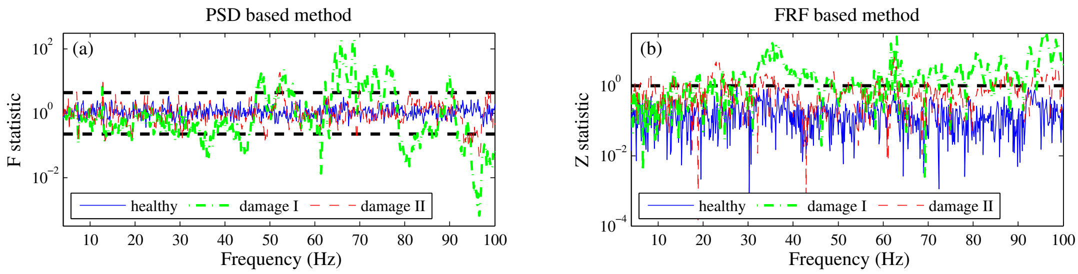

Figure 3. Non–parametric damage detection results: (a) PSD based method, and (b) FRF based method (3 test cases per method; critical points at the $\alpha = 10^{-5}$ risk level shown as dashed horizontal lines; damage is detected if the test statistic exceeds the critical points).

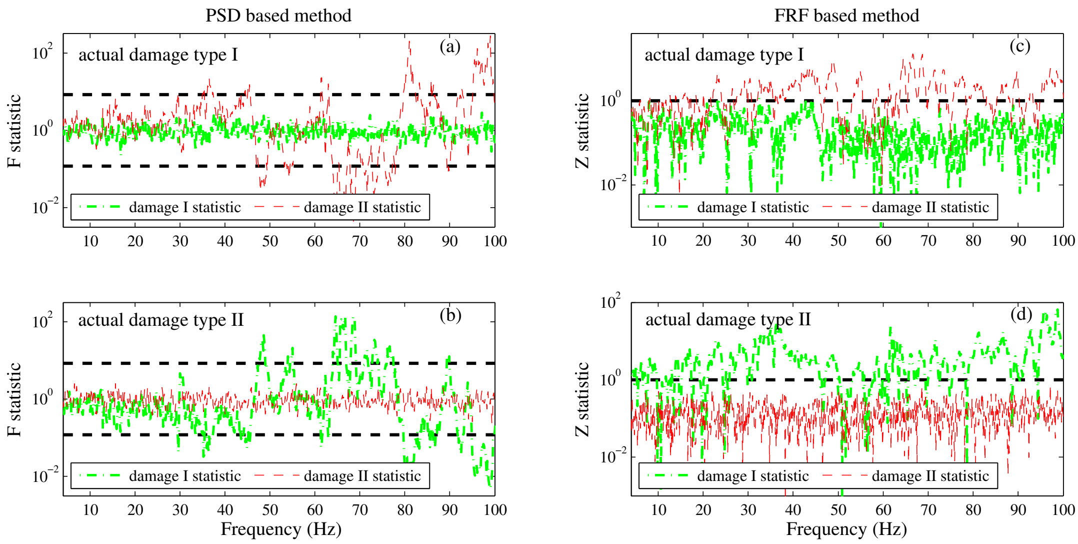

Figure 4. Non–parametric damage identification results – test statistics for the damage I and damage II hypotheses: (a) actual damage is of type I (PSD method); (b) actual damage is of type II (PSD method); (c); actual damage is of type I (FRF method); (d) actual damage is of type II (FRF method). A damage type is identified when its test statistic does not exceed the critical points (critical points at the $\alpha = 10^{-7}$ risk level shown as dashed horizontal lines).

metric methods in Table V.

Figures 3 and 4 present typical non–parametric damage detection and identification results, respectively, by using the PSD and FRF based methods. A summary of the results for all available experiments is presented in Table VI. For damage detection a single healthy data set is considered as the reference (baseline) set for the healthy structure, while the remaining ones are considered as corresponding to unknown states of the structure. Similarly, in the damage identification task, one data set for each damage type is considered as the reference (baseline) set, while the remaining ones are considered as corresponding to unknown states of the structure. As suggested by the results (Table VI), both methods achieve effective damage detection and damage type identification. The false alarm and missed damage numbers are very small, and so are the damage misclassification numbers as well. The PSD based method seems to exhibit somewhat better performance than the FRF based method,

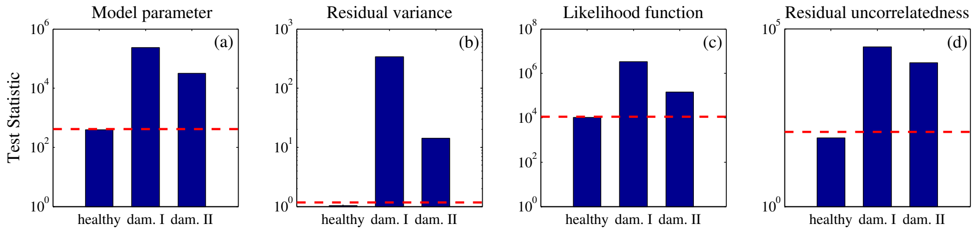

Figure 5. Parametric damage detection results: (a) model parameter, (b) residual variance, (c) likelihood function, and (d) residual uncorrelatedness based methods (3 test cases; critical points at the $\alpha = 10^{-8}$ risk level shown as dashed horizontal lines; damage is detected if the test statistic exceeds the critical point).

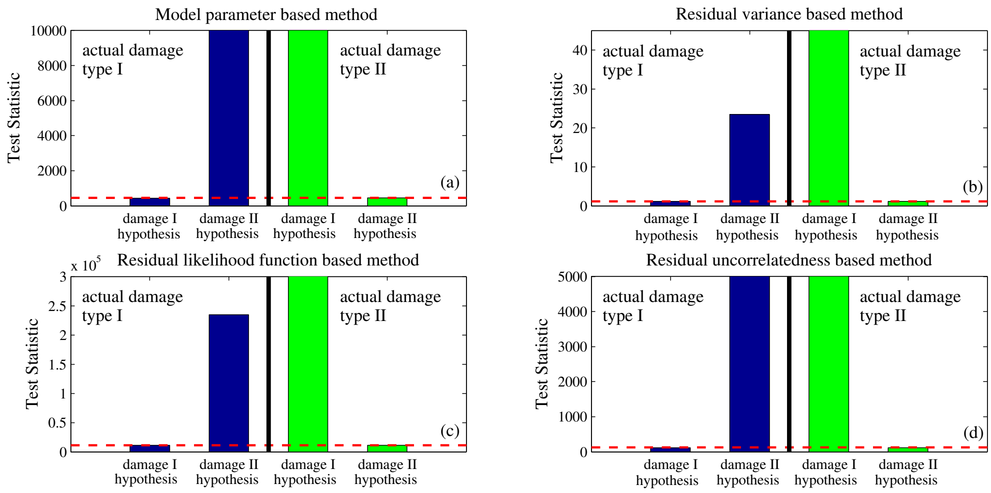

Figure 6. Parametric damage identification results – test statistics for the damage I and damage II hypotheses. Two test cases per method: actual damage I (left part), actual damage II (right part). (a) Model parameter, (b) residual variance, (c) likelihood function, and (d) residual uncorrelatedness based methods. A damage type is identified when its test statistic does not exceed the critical points (critical points at the $\alpha = 10^{-12}$ risk level shown as dashed horizontal lines).

which proved to be slightly more prone to damage misclassification.

Figures 5 and 6 present typical parametric damage detection and identification results, respectively, obtained by the model parameter, residual variance, likelihood function, and residual uncorrelatedness based methods. Notice the logarithmic scale on the vertical axis of Figure 5, which presents damage detection results by all four parametric methods. Figure 6 presents typical parametric damage identification results. The test statistics for the damage I and damage II hypotheses are shown, while two test cases per method are included: the case of actual damage of type I (left part) and actual damage of type II (right part).

TABLE VI. DAMAGE DETECTION AND IDENTIFICATION SUMMARY RESULTS

Method	Damage Detection ($\alpha = 10^{-5}$)			Damage Identification ($\alpha = 10^{-7}$)	
	False alarms	Missed damage		Damage misclassification	
		actual damage I	actual damage II	actual damage I	actual damage II
PSD based	0/39	0/32	0/32	0/31	0/31
FRF based	1/39	0/32	0/32	0/31	2/31

CONCLUDING REMARKS

An experimental assessment of non–parametric and parametric vibration–based time series methods for SHM was presented. Non–parametric methods are generally simpler, requiring little user expertise, and were shown to achieve effective damage detection and identification.

Parametric methods, on the other hand, are more elaborate and have the potential of offering increased accuracy, along with more effective tackling of the damage detection and identification subproblems. Nevertheless, accurate modelling seems necessary, while the methods may be more prone to experimental and modelling uncertainties. In this study parametric methods were somewhat more sensitive, and also exhibited increased numbers of false alarms and damage misclassifications. Overall, the results of the study confirmed the high potential and effectiveness of the vibration–based statistical time series methods for SHM.

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